लक्ष्यं विद्यमानम्

# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours Part-I Examination, 2021

## PHYSICS

## PAPER: PHSA-I

Time Allotted: 2 Hours

Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

### UNIT-IA

**Question No. 1 is compulsory and answer other questions from Group-A and Group-B according to the instructions**

1. Answer any ***two*** questions from the following: $2\frac{1}{2}\times 2 = 5$

(a) Comment on the convergence of the series $S_n = \sum_{n=1}^{\infty}(-1)^n \frac{1}{n^2}$.

(b) Check whether the function $f(z) = \sinh(z)$ is analytic.

(c) If $u\vec{a} = \vec{\nabla}v$ where $u$, $v$ are scalar fields and $\vec{a}$ is a vector field, then show that $\vec{a}\cdot\vec{\nabla}\times\vec{a} = 0$.

(d) Show that the force field $\vec{F}$ defined by

$$\vec{F} = (y^2z^3 - 6xz^2)\hat{i} + 2xyz^3\hat{j} + (3xy^2z^2 - 6x^2z)\hat{k}$$

is a conservative force field.

(e) For the matrices $A$ and $B$ prove the relation $(AB)^{-1} = B^{-1}A^{-1}$.

(f) If the probability is 0.70 that any one voter (randomly selected) will vote in an election, what is the probability that two of five voters will vote in the election?

(g) Check the singularity of $x^2y'' + (x+1)y' + ny = 0$ at $x = 0$, where '$n$' is a constant. Determine the nature of singularity, if any.

(h) 'In accelerated frames of reference, pseudo forces appear' — True or false. Explain in brief.

## GROUP-A

**Answer any *one* question from the following** $10\times1=10$

2. (a) Expand $e^x \ln(1+y)$ in powers of $x$ and $y$ in Taylor's series in the neighbourhood of $(0, 0)$ upto first six terms. 3

   (b) Find the directional derivative of $\phi(x, y, z)=3x^2y+2xz^2+4yz^2$ at the point $(1, 2, 2)$ in the direction $\hat{i}-2\hat{j}+3\hat{k}$. 2

   (c) Apply Green's theorem to evaluate $\oint_C (4xy^3\,dx+6x^2y^2\,dy)$. $C$ is the circle $x^2+y^2=1$. 3

   (d) Prove that $\delta(ax)=\frac{1}{|a|}\delta(x)$, where $\delta(x)$ is the Dirac Delta function. 2

3. (a) If $\vec{F}=\vec{r}\cdot\vec{r}$ is conservative, find out the scalar potential. 3

   (b) Find the Fourier sine and cosine transforms of $f(x)=e^{-ax}$. 2

   (c) Find the Fourier series of the following function 3+2

$$f(x)=\begin{cases} -k & \text{when } -\pi<x<0 \\ k & \text{when } 0<x<\pi \end{cases}$$

   Hence show that $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\ldots\ldots$.

4. (a) The mean of a normal (Gaussian) distribution is 50 and 5% of the values are greater than 60. Find the standard deviation of the distribution. (Given, area under standard normal curve between $z=0$ and $z=1.64$ is 0.45). 3

   (b) By using the series expansion of sine and cosine functions, show that, 4

$$\lim_{x\to0}\frac{\sin x^2}{1-\cos 2x}=\frac{1}{2}.$$

   (c) Solve $\frac{\partial^2 z(x, y)}{\partial x\,\partial y}=x^2y$, subject to the conditions $z(x, 0)=x^2$, $z(1, y)=\cos y$. 3

5. (a) Find the Fourier transform of $e^{-x^2/2}$. 3

   (b) For Legendre polynomials $P_n(x)$, show that $P_n(-x)=(-1)^n P_n(x)$. 3

   (c) The generating function for Hermite polynomials is $g(x, t)=\sum_{n=0}^{\infty}\frac{H_n(x)t^n}{n!}$. Hence 2+2

   show that

   (i) $2(n+1)H_n(x)=H'_{n+1}(x)$

   (ii) $2xH_{n+1}(x)=2(n+1)H_n(x)+H_{n+2}(x)$

6. (a) Under what conditions can a differential equation have a solution in the form of power series? 2

(b) Find the eigenvalues and normalized eigenvectors of the matrix 2+4

$$M=\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

(c) If $AB = BA$, prove that $A$ and $B$ are square matrices of same dimensions. 2

## GROUP-B

**Answer any *one* question from the following** $10\times1 = 10$

7. (a) Show that the radial and transverse components of acceleration in polar coordinates are given by $a_r = \ddot{r} - r\dot{\theta}^2$, $a_\theta = r\ddot{\theta} + 2r\dot{\theta}$. 3

(b) A particle is moving under the action of a force $\hat{K}\times\vec{V}$, where $\hat{K}$ is the unit vector along $z$-axis and $\vec{V}$ is the velocity of the particle. Show that the kinetic energy of the particle remains constant. 3

(c) Determine the principal axes and the principal moments of inertia of a cylindrical tube of length $L$, mass $M$, linear and outer radii $R_1$, $R_2$; respectively. 4

8. (a) Show that the acceleration $\vec{a}$ of a particle travelling along a curve with velocity $\vec{v}$ is given by $\vec{a} = \frac{dv}{dt}\hat{\tau} + \frac{v^2}{\rho}\hat{n}$ where $\hat{\tau}$ is the unit vector tangent to the curve, $\hat{n}$ is the unit vector normal to the curve and $\rho$ is the radius of curvature. 4

(b) A reference frame is rotating with an angular velocity $\vec{\omega}$ w.r.t. the laboratory frame. Establish the transformation relation $\frac{d}{dt} = \frac{d'}{dt} + \vec{\omega}$. How does a vector $\vec{A}$ transform in such cases if it is parallel to the axis of rotation? 3+1

(c) Two particles of masses $m_1$ and $m_2$; respectively, are connected by rigid massless rod of length '$a$' and move freely in a plane. Find the moment of inertia of the system about an axis perpendicular to the plane and passing through the centre of mass. Use the concept of reduced mass of the system $\mu = \frac{m_1 m_2}{m_1 + m_2}$. 2

## UNIT-IB

**Question No. 9 is compulsory and answer other questions from Group-C and Group-D according to the instructions**

9. Answer any ***two*** questions from the following: $2\frac{1}{2}\times2 = 5$

(a) For a homogeneous, isotropic, deformable elastic body, establish the relation $\frac{dv}{v} = \frac{dl}{l}(1-2\sigma)$ where $\sigma$= Poisson's ratio.

(b) What are 'inertial mass' and 'gravitational mass'?

(c) A circular disc of mass '$m$' and radius '$r$' is set rolling on a horizontal table. If '$\omega$' be the angular velocity of the disc, then show that its total energy is $\frac{3}{4}mr^2\omega^2$.

(d) Calculate the surface energy of a spherical liquid drop of radius $a$ and surface tension $T$.

(e) What are Newtonian and non-Newtonian fluids? Give examples.

(f) Explain 'critical damping' in case of damped oscillation.

(g) "The velocity of longitudinal wave is greater than the velocity of transverse waves in solids". — True or False. Explain in brief.

(h) Is the phase velocity and group velocity of a progressive wave equal in a non-dispersive medium? — Why?

## GROUP-C

**Answer any *one* question from the following** $10\times1 = 10$

10.(a) In case of a motion under central force, show that the areal velocity is constant. 2

(b) Prove that the total energy of a particle of mass $m$ under a central force is given by 5

$E=\frac{h^2}{2m}\left[u^2+\left(\frac{du}{d\theta}\right)^2\right]+V(r)$ where $u=\frac{1}{r}$, $h=$ angular momentum of the particle and $V(r)$ is the potential energy at $r$.

(c) The density of a sphere varies as the depth below the surface. Show that the gravitational attraction is greatest at a depth equal to $\frac{1}{3}$ of the radius. 3

11.(a) A beam of length 1 m and mass per unit length 2.0 gm/cm is kept horizontally having one end fixed. A weight of 1 kg is loaded at the free end. Find the torque at a section 40 cm apart from the fixed end. 4

(b) A horizontal steel wire of length 1 m having a cross-sectional area of 1 $mm^2$ is stretched between two fixed supports and then loaded at its mid-points. A load of mass 40 gm produces a sag in the air of 5 mm and a load of mass 600 gm produces a sag of 15 mm. Sketch the appropriate diagram and find the Young's modulus for steal. 1+3

(c) A drop of water 1 mm in diameter is broken up to a million droplets, all of the same size, calculate the work expanded in the process. Assume surface tension of water = 74 dyne/cm. 2

12.(a) A liquid having coefficient of viscosity $\eta$ flows steadily through a cylindrical tube of radius $r$ and length $l$ under pressure $P$. Show that its velocity at a distance $x$ from the axis of the tube is given by $V=\frac{P}{4\eta l}(r^2-x^2)$. 3

(b) What are the different types of elastic moduli of an isotropic solid medium? How are they related to each other? 2+2

(c) Discuss the Euler's equation of motion for a moving ideal fluid. 3

## GROUP-D

**Answer any *one* question from the following** $10 \times 1 = 10$

13.(a) Define group velocity. Derive its relation with phase velocity. 2+2

(b) Consider the general expression for standing wave for a homogeneous uniform elastic string executing simple harmonic oscillations for the following:

(i) Write the differential equation for harmonic oscillations. 2

(ii) Find the corresponding harmonics of the modes of the vibrating string and sketch the figures. 2+2

14.(a) An object of mass 2 kg hangs from a spring of negligible mass. The spring gets extended by 2.5 cm when the object is attached. The top end of the spring sets in simple harmonic motion with an amplitude of 2 mm. The θ of the system is 20. If $g = 10\ ms^{-2}$, find the following: 2+2

(i) What is the angular frequency ( $\omega_0$ ) of the free undamped oscillations?

(ii) What is the amplitude of forced oscillations at $\omega = \omega_0$ ?

(b) Two sound waves, one in air and the other in water, have the same frequency and intensity. Calculate the ratio of the pressure amplitudes of the two waves. Given, density of air = 0.00129 gm/cc, velocity of sound in air = 331 m/s and in water = 1510 m/s. 3

(c) Derive an expression for the change in frequency of a sound wave when the source and the observer are both in motion in a stationary medium. 3

15.(a) Set up the differential equation for the transverse vibration of a stretched string and solve the equation by the method of separation of variables. 2+4

(b) Derive the expression for energy density and its corresponding intensity for sound waves travelling in a gas medium. State the assumptions and validity conditions for the same. 4

**N.B. :** *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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