लक्ष्यं विश्वमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours PART-I Examinations, 2017

PHYSICS-HONOURS

PAPER-PHSA-I

Time Allotted: 4 Hours | Full Marks: 100

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.

Use separate answer scripts for each Unit.

Unit-IA

Question No. 1 is compulsory and answer other questions from Group A and B according to the instructions

1. Answer any ***five*** questions from the following: $2\times5=10$

(a) Test the condition of convergence of the harmonic series $S_n=\sum_{n=1}^{\infty}\frac{1}{n^p}$.

(b) If $u\vec{a}=\vec{\nabla}v$ where u, v are scalar fields and $\vec{a}$ is a vector field, then show that $\vec{a}.\vec{\nabla}\times\vec{a}=0$.

(c) Check whether the function $f(z)=\sin(z)$ is analytic.

(d) Assuming Poisson distribution function, calculate its mean.

(e) If a matrix A is hermitian and $A^2=1$, show that A is also unitary.

$A(A-\lambda I)=0$

$A^2-\lambda A=0$

(f) Check the singularity of $x^2y'' + (x+1)y' + ny = 0$ at $x = 0$, where 'n' is a constant. Determine the nature of singularity, if any.

(g) If a force $\vec{F}$ satisfies the relation $\vec{\nabla} \times \vec{F} = 0$, then show that the force field is conservative.

(h) A circular disc of mass M and radius r is set rolling on a horizontal table. If ω be the angular velocity of the disc, show that its total energy is $\frac{3}{4}Mr^2\omega^2$.

Group-A

Answer any *three* questions from the following $10\times3=30$

2. (a) State and develop the Taylor series for functions of two variables. 3

(b) Obtain the expression for Laplace's equations in spherical polar co-ordinates for a point in space describe by the Co-ordinates (r, θ, ϕ). 3

(c) Establish that, $\vec{\nabla} \times (\vec{\nabla} \times \vec{F}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{F}) - \nabla^2 \vec{F}$. 3

(d) What is the physical significance of $\vec{\nabla} \cdot \vec{B} = 0$? 1

3. (a) Determine an unit vector perpendicular to the plane of $\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $B = 4\hat{i} + 3\hat{j} - \hat{k}$. 3

(b) Prove that $\iiint (\phi \nabla^2 \psi - \psi \nabla^2 \phi)\, dv = \iint (\phi \vec{\nabla} \psi - \psi\ \vec{\nabla} \phi).\vec{ds}$. 3

(c) Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$. 2

(d) Prove that $\nabla^2 f(r) = \dfrac{d^2 f}{dr^2} + \dfrac{2}{r}\dfrac{df}{dr}$. 2

4. (a) Obtain the Fourier series of the periodic function (period = 2π) defined as 4+2

$$f(x) = -k \text{ for } -\pi < x \le 0$$
$$= k \text{ for } 0 < x \le \pi$$

Hence show that $\dfrac{\pi}{4} = \sum_{n=0}^{\infty} \dfrac{(-1)^n}{(2n+1)}$.

(b) Find the eigenvalues and normalized eigenvector of the matrix 2+2

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}.$$

5. (a) Comment on the singularity and type of solution for the differential equation 2

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0.$$

(b) For Legendre polynomials $P_n(x)$, show that $P_n(-x) = (-1)^n P_n(x)$. 2

(c) Show the orthogonality condition of Legendre polynomials. 2

(d) The generating function for Hermite polynomials is $g(x,t) = \sum_{n=0}^{\infty} \dfrac{H_n(x)t^n}{n!}$. 2+2

Hence show that

(i) $2(n+1)\,H_n(x) = H'_{n+1}(x)$

(ii) $2x H_{n+1}(x) = 2(n+1)H_n(x) + H_{n+2}(x)$

6. (a) Solve by the method of separation of variables 4

$$3\frac{\partial U}{\partial x} + 2\frac{\partial U}{\partial y} = 0, \quad U(x,0) = 4e^{-x}.$$

(b) If A and B are Hermitial matrix. Prove that AB is Hermitian only if A and B Commute. 2

(c) The matrix A is given by 4

$$A = \frac{1}{\sqrt{2}}\begin{bmatrix} z & z^* \\ iz & -iz^* \end{bmatrix}$$

where z is a complex number with modulus 1 and z^* is its conjugate. Show that the complex matrix A is unitary.

Group-B

Answer any *one* question from the following $10\times1 = 10$

7. (a) A particle with constant angular velocity describes the curve $r = ae^{\theta}$ where a is a constant. Show that the radial acceleration is zero and the cross radial acceleration is proportional to its distance from the pole. 3

(b) Find the corresponding potential function for 2

$$\vec{F} = (2xy - z^3)\hat{i} + x^2\hat{j} - (3xz^2 + 1)\hat{k}.$$

(c) Consider the motion of a rocket vertically upward against uniform gravity $\vec{g}$ which ejects gas with a constant velocity $\vec{u}$ with respect to itself. Obtain the equation of motion of the rocket. 2+2

If the fuel is burnt at a constant rate, calculate the maximum speed attained by the rocket.

(d) Moment of Inertia is the rotational analogue of mass of the body– Justify. 1

8. (a) Explain moment of Inertia and product of Inertia of a rigid body. 2

(b) The moment of inertia of a circular disc about an axis perpendicular to its plane and passing through its centre is $\frac{1}{2}Mr^2$. Show that the moment of inertia of the disc about an axis parallel to any diameter is $\frac{5}{4}Mr^2$. 2

(c) A reference frame is rotating with an angular velocity $\vec{\omega}$ *w. r. t.* the laboratory frame. Establish the transformation relation $\frac{d}{dt} = \frac{d'}{dt} + \vec{\omega}$. How does a vector $\vec{A}$ transform in such cases if it is parallel to the axis of rotation? 3+1

(d) Show that from Euler's equation that the rotational kinetic energy of a rigid body is conserved when the applied torque is zero. 2

Unit -IB

Question No. 9 is compulsory and answer other questions from Group C and D according to the instructions.

9. Answer any ***five*** questions from the following: 2×5 = 10

(a) For a homogeneous isotropic, deformable elastic body, establish the relation $\frac{dv}{v} = \frac{dl}{l}(1-2\sigma)$, where σ = Poisson's ratio.

(b) What are 'inertial mass' and 'gravitational mass'?

(c) Derive gravitational field intensity due to a point mass M at a distance r from it using Gauss's theorem of Gravitation.

(d) What is Raynold's number? How is it used to determine whether the nature of flow of a liquid Streamline or turbulent?

(e) Explain the molecular theory of surface tension.

(f) What are the differences between amplitude resonance and velocity resonance?

(g) Define 'Bel'. Show that a reduction in amplitude by same factor reduces the sound intensity.

(h) Define co-efficient of viscosity. What is meant by Non-Newtonian liquids?

Group-C

Answer any *two* questions from the following 10×2 = 20

10.(a) Prove that the total energy of a particle of mass m under a central force is given by 5

$$E = \frac{h^2}{2m}\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right] + V(r)$$

where $u = \frac{1}{r}$, h = angular momentum of the particle and $V(r)$ is the potential energy at r.

(b) The density of a sphere varies as the depth below the surface. Show that the gravitational attraction is greatest at a depth equal to $\frac{1}{3}$ of the radius. 3

(c) What do you understand by gravitational self energy of a homogeneous sphere? 2

11.(a) State the basic assumptions for flow of a liquid through a narrow horizontal tube and derive the Poiseuille's formula. 2+4

(b) Show that the excess pressure acting on the curve surface of a membrane is given by $P = 2S\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$, where r_1 and r_2 are the radius of curvature and S is the surface tension of the membrane. Draw appropriate figures. 3+1

12.(a) What is a cantilever? Find the depression of the free end of a cantilever of uniform circular cross-section when loaded by weight 'w' considering its own weight w_0. 1+3

(b) A liquid having coefficient of viscosity η flows steadily through a cylindrical tube of radius r and length l under pressure P. Show that its velocity at a distance x from the axis of the tube is given by $V = \frac{P}{4\eta l}(r^2 - x^2)$. 3

(c) Applying Stoke's Law for the motion of a spherical body through a viscous medium find out an expression of the terminal velocity of the body. 3

Group-D

Answer any *two* questions from the following $10\times2 = 20$

13.(a) Two mutually perpendicular simple harmonic motions are acted simultaneously on a particle. The motions are represented by $x = a \sin \omega t$ and $y = b\sin(\omega t+\delta)$. Find out the equations of the combined motion. Mention the condition when the locus of the particle will be a straight line with gradient = 1. 3+1

(b) Consider forced vibration in the steady state. Obtain an expression for the average power supplied by the external periodic force over a period. Show that it is equal to the power dissipated to overcome the damping forces. In this connection explain power factor. What is its value at resonance? 2+2+1+1

14.(a) Show that the general solution of the wave equation $\dfrac{\partial^2 y}{\partial t^2} = v^2 \dfrac{\partial^2 y}{\partial x^2}$ may be written as $y = f(x-vt)+g(x+vt)$. 3

(b) For a plane, progressive wave, show that the instantaneous energy density is not constant but its average value over a complete period is constant. 4

(c) Derive an expression for the velocity of a plane longitudinal wave in a fluid medium. 3

15.(a) A stretched string of length l and fixed at the ends is plucked at the mid-point through a distance h. Find an expression for the displacement at a point x on the string at time t. 2

(b) For a struck string the displacement at a point x at time t can be represented by a superposition of harmonics denoted by s, as follows: 1+2

$$y(x,t) = A \sum_{s=1,2,3} \frac{1}{s} \sin\frac{s\pi a}{l} \sin\frac{s\pi x}{l} \sin\frac{s\pi ct}{l}$$

where l is the length of the string, a is the point of striking, A is a constant and c is the velocity of transverse wave through the string.

(i) Find the nodal points of the s-th harmonics.

(ii) Show that if the point of striking coincide with any of the nodal points of s-th harmonics; it will be absent from the vibration.

(c) Two horns are blowing with equal frequency of 350 Hz at two different places P and Q. A person is moving straight towards Q from P with a velocity of 9 km/hr and hears 5 beats. Determine the velocity of sound. 3

(d) What do you mean by normal modes and normal co-ordinates in connection with coupled vibration? 2