B.Sc./Part-I/Hons/PHSA-I/2016

2016

30

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours PART-I Examinations, 2016

PHYSICS-HONOURS

PAPER-PHSA-I

Time Allotted: 4 Hours — Full Marks: 100

The figures in the margin indicate full marks. Candidates should answer in their own words and adhere to the word limit as practicable.

Q. No. 1 is compulsory and answer other questions from Group A, B, C and D according to the given instructions under each group.

1. Answer any *ten* questions from the following: $2\times10=20$

(a) Show that $f(z)=e^z$ is analytic in the entire complex plane.

(b) If $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}-3\hat{k}$ and $3\hat{i}+a\hat{j}+5\hat{k}$ are coplanar, find a.

(c) For the matrices A and B prove the relation, $(AB)^{-1}=B^{-1}A^{-1}$. → 64 (AB)

(d) Let the solution of two dimensional Laplace's equation in the cartesian coordinate, i.e., $u(x,y)$ be a function of a single variable P. Assuming $P=x+\lambda y$, show that $\lambda^2+1=0$.

(e) Using generating function of the Hermite polynomial, show that $2nH_{n-1}(x)=H_n'(x)$.

(f) Prove that $\delta(ax)=\frac{1}{|a|}\delta(x)$, where $\delta(x)$ is the Dirac delta function.

(g) 'In accelerated frames of reference, pseudo forces do appear'-Justify.

(h) A circular disc of mass M and radius R is rolling without slipping on a horizontal surface with speed V. Find its kinetic energy.

(i) What are half power frequencies? How are they related to sharpness of resonance?

(j) What is an ideal fluid? Define the stream line flow of an ideal fluid.

(k) Define the angle of contact. Indicate with figures the conditions for the angle of contact to be acute or obtuse.

(l) State the reciprocity theorem for an elastic cantiliver.

(m) The length of a wire is l_1 when tension is T_1 and is l_2 when the tension T_2. What is the natural length of the wire?

(n) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{2^n}{n^3}$. 8

(o) Show that for a nondispersive medium, the group velocity and the phase velocity are equal.

(p) Write down the displacement for a plane harmonic progressive wave with frequency 300 Hz and velocity 340 mt/sec along a direction having direction cosines $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6}}\right)$.

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix} \quad \frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}$$

Group-A

Answer any *three* questions from the following: $10\times3=30$

2\. (a) Prove the identity, $\nabla\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{A}) - \vec{A}\cdot(\nabla\times\vec{B})$. Hence, show that $(\vec{A}\times\vec{r})$ is solenoidal if $\vec{A}$ is irrotational. $4+3+3$

(b) Prove the vector integral theorem, $\int \vec{\nabla}\phi\, dV = \int \phi\, d\vec{S}$, where $\phi(\vec{r})$ denotes a well behaved scalar field and $\vec{S}$ is the closed surface enclosing a volume V.

(c) If $f(x,y,z)=0$, then show that $\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$.

3. (a) Let $\hat{r}$, $\hat{\theta}$ and $\hat{\phi}$ be the unit vectors corresponding to the spherical polar coordinates. Show that the differential displacement vector is given by, $d\bar{r} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta\, d\phi\hat{\phi}$. Hence determine the differential area elements with normal along $\hat{r}$, $\hat{\theta}$ and $\hat{\phi}$. Prove that the differential volume element is $dV = r^2 \sin\theta\, dr d\theta\, d\phi$. (4+3+1)+2

(b) If matrix A commutes with matrix B, prove that A also commutes with B^{-1}. — 64(AB)

4. (a) Show that the Fourier series expansion of an odd function contain only sine terms. 3+3+4

(b) State Dirichlet's conditions for the convergence of the Fourier series expansion of periodic functions, explaining any technical terms you use.

(c) Express $\dfrac{\partial^2}{\partial x^2}$ in the plane polar coordinate system.

5. (a) Solve by separation of variable method the partial differential equation $\nabla^2\psi = \dfrac{1}{C^2}\dfrac{\partial^2\psi}{\partial t^2}$, in plane polar coordinates. 4+3+3

(b) Calculate a particular Integral for the equation, $\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} + 4y = \cos 2x$.

(c) Establish the orthogonality condition for the Hermite polynomial. → 313

6. (a) Solve the linear harmonic oscillator equation, $\dfrac{d^2y}{dx^2} + \omega^2 y = 0$ by the power series substitution method, about $x = 0$. 6+(2+2)

3

(b) Establish the following recurrence relations using generating function for the Legendre polynomial,

(i) $P'_{n+1}(x) = (n+1)P_n(x) + xP'_n(x)$,

(ii) $(1-x^2)P'_n(x) = nP_{n-1}(x) - nxP_n(x)$.

Where $P_n(x)$ is the Legendre Polynomials of order n.

Group-B

Answer any *one* question from the following: 10×1 = 1(

7. (a) Write down the Galilean transformations and show that Newton's second law of motion is invariant under them. 3+4+:

   (b) 'The motion of the particle is completely determined by the equation of motion if the position and velocity are specified at the initial instant of time.'—Explain.

   (c) Establish that, the total mechanical energy of a particle moving in a conservative field remains constant.

8. (a) Two masses m_1 and m_2 are moving under mutual interaction only. Show that the motion can be viewed as two separate motions, viz., the free motion of the centre of mass and the motion of a single particle with reduced mass. 5+2+

   (b) While studying the motion of the Earth-Sun two-body system under gravitation, we consider the Earth to be moving around the fixed Sun. Justify mathematically.

   (c) A cylinder of radius R has mass distribution that varies linearly with the distance from its axis. Find the moment of inertia of the body of mass M about its axis.

Group-C

Answer any *two* questions from the following: 10×2 = 2

9. (a) Determine the gravitational potential and the field at a point (i) outside, (ii) inside and (iii) on the surface of a spherical shell. 5+3+

   (b) Derive an expression for the equation of continuity for a fluid in motion.

   (c) A horizontal tapered tube of circular cross-section has radii 0.5 and 0.3 cm at two ends. The pressure difference at two places is 1 cm of water. Calculate the rate of flow through the tube.

10.(a) Show that the strain energy of a twisted wire is $\frac{1}{2}C_m\theta_m$ where C_m is the couple for the maximum twist θ_m. 3+(3+2)+2

(b) State and explain Bernoulli's principle. Why it is dangerous to stand close to the railway track when a Fast train is passing.

(c) Two spherical soap bubbles having radii 3 cm and 4 cm respectively coalesce so as to have a part of their surface common. What is the radius of curvature of the common surface?

11.(a) Deduce the following relation $Y = 3K(1-2\sigma)$, where symbols have their usual meaning. 3+2+2+3

(b) 'A liquid film when adiabatically stretched gets cooled'-Explain.

(c) With proper assumptions, state the Stokes law of viscous force acting on a spherical body as it descends through a viscous fluid.

(d) Show that a planet revolving around the Sun in an elliptical orbit, keeping the Sun at one of its foci, is subjected to an inverse square force field.

Group-D

Answer any *two* questions from the following: 10×2 = 20

12.(a) Set up the equation of motion of a simple harmonic oscillator subject to (1+3)+3+3

(i) a damping force proportional to velocity and

(ii) an external sinusoidal force .

Now solve the equation.

(b) Derive the condition for the amplitude resonance for the above case.

(c) Find an expression for the energy density of a plane progressive wave.

13.(a) Show that the velocity of acoustic waves travelling along a solid rod is given by $\sqrt{\frac{Y}{d}}$, where Y is the Youngs modulus and d is the density. Mention the assumptions made. (4+1

(b) What do you mean by intensity level and sound pressure level? Define Bel. The intensity of a sound is 10^{-2} Watt/m^2 at a frequency of 1 kHz. What is its loudness level? Use the reference intensity level as 10^{-12} Watt/m^2. (2+1+

14.(a) For a stretched string of length L the displacement is given by, 4+2+(1+

$$Y(x,t) = \sum_n C_n \sin\left(\frac{n\pi x}{L}\right)\cos(\omega_n t - \delta_n),$$

Where the symbols have their usual significance. Show that the total energy of the string is, $E = \frac{M}{4}\sum_n \omega_n^2 C_n^2$, where M is the mass of the string.

(b) Consider two strings of same length and same material. Tensions in the two strings are in the ratio 4 : 1 and diameters are in the ratio 1 : 2. Compare the frequencies of the fundamental modes of the vibration.

(c) A prominent spectral line of a certain substance has wavelength 5000 Å in the laboratory. The same spectral line is identified in the light coming from a distant star but with a different wavelength 5001Å. Is the star receding from or moving towards the earth? Calculate its speed with respect to the earth.

$\nu = \frac{1}{2l}\sqrt{T/m}$

$= \frac{1}{2l}\sqrt{\frac{T}{\pi (D/2)^2 \rho}}$

$\frac{\nu_1}{\nu_2} = \sqrt{\frac{T_1}{D_1^2}\frac{D_2^2}{T_2}}$

$= \sqrt{\frac{T_1}{T_2}\times\frac{D_2^2}{D_1^2}}$

$= \sqrt{\frac{4}{1}\times\frac{4}{1}} = 4/1$

$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

$= \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \cdot \rho}}$

$n \propto \frac{\sqrt{T}}{r}$