2015

PHSA (HN)-01

West Bengal State University

B.A./B.Sc./B.Com (Honours, Major, General) Examinations, 2015

PART – I

PHYSICS — HONOURS

Paper – I

Duration : 4 Hours] [Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

1. Answer any *ten* of the following questions : $10 \times 2 = 20$

a) Is the series $1 - \frac{1}{2} + \frac{1}{8}$ ..... absolutely convergent, convergent or divergent ?

b) Assuming Poisson distribution function, calculate its mean.

c) State Green's theorem in a plane.

d) Prove that a Hermitian matrix has real eigenvalues.

e) Can e^x and tan x be expressed as Fourier series ?

f) The position vector of a particle moving in $X - Y$ plane is given by $\vec{r} = 4t\,\hat{i} + (8t - 2t^2)\,\hat{j}$, where t is time in seconds. At what time will the velocity and acceleration be perpendicular to each other ?

g) Prove that Kepler's first and second laws lead to the conservation of angular momentum.

h) Starting from the relations among the elastic coefficients and Poisson's ratio, obtain the limitting values of Poisson's ratio.

i) What is the value of the moment of inertia of a right circular cylinder when its length is equal to its radius ?

j) A cylindrical vessel of mass 500 gm and outer radius 10 cm is floating in water. It has a circular hole of radius 0·2 mm at the bottom surface. Calculate the maximum weight that can be placed in the vessel without submerging it. Surface tension of the water is 73 dynes/cm.

k) A tube of radius R and length L is connected with another tube of radius $R/2$ and length $L/4$. If the pressure across the two tubes is p, then calculate the pressure across the tubes separately.

l) What do you mean by saturated vapour pressure at a particular temperature ?

m) A particle at a time t has displacement of the type $x = a \cos \omega t + b \sin \omega t$. Is the motion of the particle simple harmonic in nature ? Why ?

n) Distinguish between velocity resonance and amplitude resonance.

o) What is the difference between forced vibration and coupled oscillation ?

p) For free paths of length x during which a molecule does not suffer a collision with another molecule in a dilute gas, one uses the exponential distribution :

$$P_E(x:\lambda) = 1/\lambda\, e^{-x/\lambda}\ ;\ 0 \le x \le \infty .$$

Show that the average value of x is λ in the above distribution.

UNIT - I (A)

Group - A

Answer any *three* questions. $3 \times 10 = 30$

2. a) Expand $e^x \ ln\ (1 + y)$ in powers of x and y in Taylor's series in the neighbourhood of $(0, 0)$ up to first six terms. 3

b) Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 1 & -2 \\ -2 & -2 \end{bmatrix}.$$ 4

c) If $\phi \vec{A}$ is irrotational, but $\vec{A}$ is not, then show that $\vec{\nabla} \times \vec{A}$ is perpendicular to $\vec{A}$ except at points where ϕ vanishes. 3

3. a) If $A = (y + 2x)\hat{i} + (3x + 2y)\hat{j}$, find the line integral of A about a circle in xy plane with centre at the origin and radius 2, if C is traversed in the positive direction. 3

b) Consider a vector A which is a function of a parameter t. Show that

$$d\vec{A}/dt = (dA_\rho/dt - A_\phi d\phi/dt)\hat{\rho} + (dA_\phi/dt + A_\rho\, d\phi/dt)\hat{\phi} + dA_z/dt\,\hat{z}$$

where A_ρ, A_ϕ and A_z are the components of $\vec{A}$ in cylindrical polar co-ordinates. 4

c) If $f(x, y, z) = 0$, then show that $\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$. 3

4. a) Obtain the Fourier transform of Gaussian distribution function given by $e^{-\alpha^2 t^2}$. 4

b) If the solution $y(x)$ of Hermite differential equation is written as

$$Y(x) = \sum_{r=0}^{\infty} a_r x^{k+r},$$

then show that the allowed values of k are zero and one only. 4

c) Write down the orthogonality property of Legendre polynomial. 2

5. a) Using Green's theorem for a plane calculate area of an ellipse

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1.$$ 2

b) Is the matrix $[A] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$ unitary ? 2

c) If a matrix A commutes with a diagonal matrix B, no two elements of which are equal, show that A is a diagonal matrix. 2

d) Solve by the method of separation of variables :

$$3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0, \; u(x, 0) = 4e^{-x}.$$ 4

6. a) The Gaussian probability distribution function is given by

$$P_G(x : \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right], \quad -\infty < x < \infty$$

i) Show that it has two point of inflexions at $x = \mu \pm \sigma$

ii) What is the area under the curve ? What does it signify ? 2 + 2

b) Expand the function $f(x) = |x|$, $-l < x < l$ in a Fourier series. 4

c) How are the original function and its Fourier expansion related outside the range when (i) the function is non-periodic, (ii) periodic ? 2

Group - B

Answer any *one* question. $1 \times 10 = 10$

7. a) Write down Gallilean transformation relations in matrix form. Show that under Gallilean transformation Newton's second law of motion remains invariant. 2 + 2

b) A particle is moving under the action of a force $\hat{K} \times \vec{V}$, where $\hat{K}$ is unit vector along Z-axis and $\vec{V}$ is the velocity of the particle. Show that the kinetic energy of the particle remains constant. 3

c) Obtain the expressions for the radial and transverse components of acceleration in plane polar co-ordinate system for the motion of a particle in a plane. 3

8. a) Show that the rate of change of total angular momentum of a system of particles equals the total torque acting on the system. 3

b) What do you mean by principal axes of a rigid body ? Find the principal axes and principal moments of inertia of a cylindrical tube of length L, mass M, inner and outer radii R_1 and R_2 respectively. 1 + 4

c) What will be the shape of the ellipsoid of inertia for a sphere, with origin at the centre ? 2

UNIT - I (B)

Group - C

Answer any *two* questions. $2 \times 10 = 20$

9. a) A particle is acted upon by a central force describes an orbit given by $r = a(1 + \cos\theta)$, a being a constant. Show diagrammatically the nature of the orbit and find the nature of the force. 3

b) The maximum angular speed of a planet is n times its minimum angular speed. Show that the eccentricity of the orbit of the planet is $\frac{(\sqrt{n}-1)}{(\sqrt{n}+1)}$. 3

c) Define gravitational self energy. Imagine that a Galaxy consists of n stars each of mass m with an average separation r between every pair of them, *i.e.* $r_{ij} = r$ for all i and j. Calculate the gravitational self energy of the Galaxy.

(Ignore gravitational self energy of the individual stars).

Given $n = 1{\cdot}6 \times 10^{11}$, $m = 10^{23}$ gms, $r = 10^{23}$ cms,

$G = 6{\cdot}673 \times 10^{-8}$ cm^3 / gm-s^2. 1 + 3

10. a) Show that within the elastic limit the depression at a point P in a loaded uniform beam due to a load F applied at another point Q is the same as the depression at Q due to the same force F applied at P. 4

b) Consider a rod of circular cross-section, of length l and radius a. The volume of the rod is not changed when the rod is stretched, show that the Poisson's ratio of the material of the rod is $\frac{1}{2}$. 2

c) Deduce the Poiseuille's formula for the flow of a viscous fluid through a narrow horizontal capillary tube mentioning the necessary assumptions made. 1 + 3

11. a) Show that the excess pressure acting on the curved surface of a membrane is given by $P = 2S\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$, where r_1 and r_2 are the radii of curvature and S is the surface tension of the membrane. 3

b) A fluid is at rest under the action of an external force F and a pressure gradient force. If the pressure be p at a point (x, y, z), show that $\vec{F} = \vec{\nabla} p$. 3

c) A horizontal tube has radii 0·5 cm and 0·3 cm at two places. For flowing water, the pressure difference at the two places is 1 cm of water. Calculate the rate of flow of water through the tube. 2

d) If the rate of change of surface energy of a liquid with temperature is directly proportional to the absolute temperature T, show that the surface tension $S = AT^2 + BT + C$, where A, B and C are constants. 2

Group - D

Answer any *two* questions. $2 \times 10 = 20$

12. a) A horizontally mounted spring stretches 0·03 m when subjected to a force of 6N. A 0·5 kg mass is attached to the free end and given an initial displacement of 0·015 m and an initial velocity of 0·4 m/s. Find the time period, and amplitude qualitatively of the motion. 2 + 2

b) Set up the differential equation of motion for a damped simple harmonic oscillator. Assuming the motion starts at $t = 0$ from $x = A$ with zero velocity, solve the equation for low damping. 1 + 3

c) Show that the velocity c of sound in a gas is given by $c = \left(\frac{\gamma}{3}\right)^{1/2} v_r$, where v_r is the r.m.s. speed of the gas molecules and γ is the ratio of the two specific heats. 2

13. a) Show that a uniform circular motion is equivalent to two mutually perpendicular simple harmonic motions of the same frequency but with a phase difference of $\frac{\pi}{2}$. 2

b) Also show that a simple harmonic motion is equivalent to two oppositely moving circular motions. 2

c) A flexible string of uniform cross-section and length l along x-axis is fixed at two ends under a tension T. Set up the differential equation for transverse vibration of the string. What will be the fundamental frequency for the string ? 4 + 2

14. a) Derive an expression of longitudinal velocity of progressive wave in gaseous medium. 4

b) Two plane progressive waves of same frequency, amplitude and wavelength are moving in opposite directions. Find the wavelength of the wave if the amplitude of the resultant motion is zero at points x_1, x_2, ... , where $x_2 - x_1 = 5$ cm. Derive the formula you use. 4

c) Distinguish between stationary wave and progressive wave. 2