West Bengal State University

B.A./B.Sc./B.Com (Honours, Major, General) Examinations, 2014

PART - I

PHYSICS — HONOURS

Paper - I

Duration : 4 Hours] [Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

UNIT - I

1. Answer any *ten* of the following questions : $10 \times 2 = 20$

a) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$.

b) Find the unit vector perpendicular to $\vec{A} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{B} = -2\hat{i} + \hat{j} - 2\hat{k}$.

c) Check the singularity of $xy'' + (1 - x) y' + ay = 0$ at $x = 0$, where a is a constant. Determine the nature of singularity, if any.

d) A particle of mass m moves in the xy plane so that its instantaneous position vector is given by $\vec{r} = a \cos \omega t \ \hat{i} + b \sin \omega t \ \hat{j}$, where a, b and ω are constants. Show that the force acting on the particle is always directed towards the origin.

e) Show that the trace of a matrix remains invarient under similarity transformation.

f) Obtain the Fourier transform of a function $f(t) = e^{-\alpha t^2}$, (α = const.)

Given $\int_{-\infty}^{+\infty} e^{-\alpha x^2 - \beta x}\, dx = \sqrt{\frac{\pi}{\alpha}}\, e^{\beta^2/4\alpha}$.

g) State two corrections required in practical situation to the Poiseuille's equation.

h) Define 'Bel'. Show that a reduction of amplitude by a factor $\frac{1}{10}$ reduces the intensity by 20 dB.

i) A circular disc of mass M and radius r is set rolling on a table. If ω be the angular velocity, show that its total energy is $\frac{3}{4} Mr^2\omega^2$.

j) A vessel has a hole of radius r at its bottom. Show that a liquid kept inside will come out of vessel it its depth exceeds $h = \frac{2T}{\rho g r}$, where T is surface tension and symbols have their usual meaning.

k) Prove that if the total momentum of a system is conserved, then the centre of mass is either at rest or in uniform motion.

l) Find whether $d\varphi$ is an exact differential where

$$d\varphi = \left(x^3 - 4xy\right) dx + \left(y^2 - 2x\right) dy.$$

m) Prove that $\oint \vec{r} \times d\vec{s} = 0$; where notations have usual meanings.

n) Write down the equation of continuity for the flow of a fluid. What happens to the equation when the flow is steady and the fluid is incompressible ?

o) Explain why large raindrops come down very fast.

p) The radius of the earth decreases by 1%, but its mass remains unchanged. Will g on the earth's surface increase or decrease ? And by what per cent ?

UNIT - I (A)

Group - A

Answer any *three* questions. $3 \times 10 = 30$

2. a) Write down the expression of Binomial distribution function for a random variable x. Calculate mean and variance from this distribution. 1 + 2 + 2

b) Prove the identity $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$. 3

c) Find out the unit vector normal to the surface $x^2 y + 2xz = 4$ at the point $(2, -2, 3)$. 2

3. a) Calculate $\frac{\partial}{\partial x}$ in spherical polar co-ordinate system. 3

b) Evaluate $\oint \vec{A} \cdot d\vec{r}$ around the right angled triangle PQR having its vertices $P(0, 0)$, $Q(2, 0)$ and $R(2, 1)$ and being right angled at Q, where $\vec{A}$ is given by $\vec{A} = \hat{i}\left(2x + y^2\right) + \hat{j}(3y - 4x)$. 3

c) An analytic function $f(z)$ has real part $e^{-x}(x\cos y + y\sin y)$ and $f(0) = 1$. Show that $f(z) = 1 + ze^{-z}$. 4

4. a) Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$ 2 + 2

b) If a matrix is both Hermitian and unitary, show that all its eigenvalues are ± 1. 3

c) Prove that the series $x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots$ is convergent for $x < 1$, and divergent for $x > 1$. 3

5. a) Legendre polynomials may be expressed as

$$\left(1 - 2xt + t^2\right)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)\, t^n$$

Use the above relation to show that

i) $nP_n(x) = (2n-1)\, x P_{n-1}(x) - (n-1) P_{n-2}(x)$

ii) $(2n+1) P_n(x) = P'_{n+1}(x) - P'_{n-1}(x)$. 2 + 2

b) Find the Fourier series expansion for the function $f(x)$ defined by

$f(x) = x$, for $0 < x < \pi$ and $f(x) = -x$ for $-\pi < x < 0$ with

$f(x + 2\pi) = f(x)$. 4

c) Prove that the area bounded by a closed curve C in $X - Y$ plane is given by

$$\frac{1}{2} \oint_C (x\,dy - y\,dx)$$ 2

6. a) A Cartesian coordinate system is rotated about the Z-axis in anti-clockwise direction through an angle θ. Find the relation between the new coordinates x', y', z' and old coordinates x, y, z. 3

b) Prove $\nabla^2 f(r) = \dfrac{d^2 f}{dr^2} + \dfrac{2}{r}\dfrac{df}{dr}$. 3

c) Solve the equation by power series method around the point $x = 0$:

$$\left(1 - x^2\right)\frac{d^2 y}{dx^2} - 2x\frac{dy}{dx} + 6y = 0$$ 4

Group - B

Answer any *one* question. $1 \times 10 = 10$

7. a) Prove that the force field $\vec{F} = (yz - y)\hat{i} + (xz - x - 1)\hat{j} + (xy - 2z)\hat{k}$ is conservative and find the corresponding potential function. 5

b) Prove that for a system of particles, the total angular momentum about any point is the sum of the angular momentum about that point of the total mass located at its centre of mass and the angular momenta of the individual particles about the centre of mass. 3

c) A particle of unit mass moves according to the equation $\vec{r} = \hat{i}(2 + 3t^2) + \hat{j}\,5t^2 + \hat{k}t$. Find the torque $\vec{N}$ acting on the particle about the origin. 2

8. a) What do you mean by 'Coriolis force' ? Calculate its action on a vertically falling body. 1 + 3

b) The moments of inertia of a rigid body are given by I_{xx}, I_{yy}, I_{zz} and products of inertia I_{xy}, I_{yz} and I_{zx}. Find out the moment of inertia of the body about an axis whose direction cosines are given by *l, m, n*. 3

c) Calculate the principal moments of inertia of a uniform square lamina at its centre. 3

UNIT - I (B)

Group - C

Answer any *two* questions. $2 \times 10 = 20$

9. a) Show that the differential equation of motion of a particle under the influence of a central force $F(r)$ can be written as

$$\frac{d^2u}{d\theta^2} + u = \frac{-m}{L^2u^2} F\left(\frac{1}{u}\right)$$

where $u = \frac{1}{r}$, L is the angular momentum and m be the mass of the particle. 4

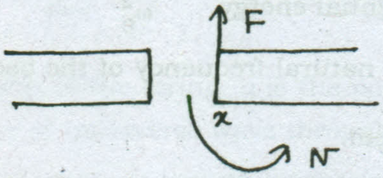
b) Find the Gravitational potential of a circular disc of uniform surface density of mass σ and radius R at a point on its circumference. 3

c) Write down Gauss' law of gravitation. Apply it to obtain an expression for gravitational field intensity at an external point for a uniform thin spherical shell. 1 + 2

10. a) A liquid of coefficient of viscosity η flows steadily through a cylindrical tube of radius r and length l under pressure P. Show that its velocity at a point inside the tube situated at a distance x from the axis is given by

$$V = \frac{P}{4\eta l}\left(r^2 - x^2\right).$$ 4

b) A homogeneous beam of uniform cross-section has a weight w per unit length, over any cross-section at x as shown in the figure. The shearing force is F and internal bending moment is N.



Establish the following relation.

i) $w = -\frac{dF}{dx}$ ii) $F = -\frac{dN}{dx}$. 2 + 2

c) Why does water rises in a capillary tube ? 2

11. a) Two soap bubbles of radii a and b coalesce to form a single bubble of radius c. If the external pressure is P, show that the surface tension is given by

$$\frac{P}{4}\frac{\left(c^3 - a^3 - b^3\right)}{\left(a^2 + b^2 - c^2\right)}$$ 4

b) State Bernoulli's theorem for fluid dynamics. Apply it to obtain Torricelli's expression for velocity of efflux of a fluid. 1 + 2

c) Applying Stokes' law for the motion of a spherical body through a viscous medium find out an expression of the terminal velocity of the body. 3

Group - D

Answer any *two* questions. $2 \times 10 = 20$

12. a) The displacement of a particle of mass m executing underdamped simple harmonic motion is given by

$$x(t) = Ae^{-bt} \cos(\omega t - \varphi)$$

where $\omega = \sqrt{\omega_0^2 - b^2}$, ω_0 = natural frequency and b = damping factor.

Calculate A and φ subject to the initial conditions at $t = 0$, $x(t) = 0$ and $\dot{x}(t) = v_0$. Hence show that in this situation, the energy of the oscillator is

$$E(t) = \frac{1}{2} m v_0^2 e^{-2bt}.$$ 2 + 3

b) Show that in forced vibration

$$\frac{\text{average kinetic enegy}}{\text{average potential energy}} = \frac{\omega^2}{\omega_0^2}$$

where ω_0 is the natural frequency of the oscillator and ω is the frequency of the forcing system. 3

c) A particle vibrates harmonically at a frequency 0·25 Hz. At the initial moment it is in the equilibrium position ($x = 0$) moving along the + *ve* x-axis at a speed of 10 cm/sec. Write down its displacement as a function of time. 2

13. a) For a one-dimensional plane progressive wave moving along (+ *ve*) X-axis, show that average enenegy density is independent of time. 4

b) The phase velocity of a surface wave on a liquid of density ρ and surface tension T is given by

$$V_p = \left(\frac{g\lambda}{2\pi} + \frac{2\pi T}{\lambda\rho} \right)^{1/2}$$

where λ is the wavelength of the wave and g is the acceleration due to gravity. Find the group velocity of the surface wave. 2

c) Consider two cases (i) a source approaching a stationary observer with a relative velocity v and (ii) an observer is approaching a stationary source with the same relative velocity v. Will the Doppler shift be same in the two cases ? 2

d) A uniform stretched string of length 1 m and mass 1 gm is under tension T. The string vibrates in three segments at a frequency of 500 Hz. Find the tension T. 2

14\. a) A stretched string of length l and fixed at the ends is plucked at the mid-point through a distance h. Find an expression for the displacement at a point x on the string at time t. 3

b) For a struck string, the displacement at a point x at a time t can be represented by a superposition of harmonics, denoted by s, as follows :

$$y(x,t) = A \sum_{s=1,2,3,\ldots} \frac{1}{s} \sin \frac{s\pi a}{l} \sin \frac{s\pi x}{l} \sin \frac{s\pi ct}{l}$$

where l, is the length of the string, a is the point of striking, A is a constant and c is the velocity of transverse wave through the string.

i) Find the nodal points of the sth harmonics.

ii) Show that if the point of striking coincide with any of the nodal points of the sth harmonics, it will be absent from the vibration. 1 + 2

c) What is the length of the Nickel rod, clamped at the middle, in a magnetostrictive oscillator generating ultrasonic waves of frequency 25 kHz ? Given Young's Modulus of Nickel = $20{\cdot}6 \times 10^{10}$ N/m^2 and mass density = 8900 kg/m^3. 2

d) What do you mean by normal modes and normal co-ordinates in connection with coupled vibration ? 2