WEST BENGAL STATE UNIVERSITY

B.Sc. Honours Part-I Examination, 2021

MATHEMATICS

PAPER: MTMA-I

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

GROUP-A

Answer any *three* questions from the following $5\times3 = 15$

1. (a) Let a and b be two positive integers such that G.C.D of $a, b = (a, b) = 1$. Prove that $(ac, b) = (c, b)$. 2

   (b) Let $a > 1$ and m, n are positive integers. Prove that $(a^m - 1, a^n - 1) = a^{(m,n)} - 1$. 3

2. If p is prime then prove that there exist no positive integers a and b such that $a^2 = pb^2$. 5

3. Solve: $111x \equiv 75 \pmod{321}$. 5

4. (a) Expand $\cos 5\theta$ in powers of $\cos\theta$. 2

   (b) Find all the values of $(-i)^{\frac{3}{4}}$. 3

5. Express i^i in the form $a + ib$, with a, b reals. Use it to find the value of $\sin(\log i^i)$ in integer. 2+3

6. If $z = x + iy$ then prove that $|\sinh y| \le \max(|\sin z|, |\cos z|) \le \cosh y$. 5

7. (a) Find $f(x-3)$ if $f(x-2) = 4x^4 + 3x^2 - x + 2$. 2

   (b) If the expression $x^3 + 3px^2 + 6qx + r$ and $x^2 + 2px + 2q$ have a common factor, show that $4(p^2 - 2q)(4q^2 - pr) = (r - 2pq)^2$. 3

8. If a, b, c, are roots of $x^3 + qx + r = 0$, form the equation whose roots are $\frac{b^2+c^2}{a^2}$, $\frac{a^2+c^2}{b^2}$, $\frac{b^2+a^2}{c^2}$. 5

9. Use Strum's function to show that the roots of the equation $x^4 + 5x^3 - 13x + 5 = 0$ are all real and distinct. 5

GROUP-B

Answer any *one* question from the following $10\times 1 = 10$

10. (a) Let A, B, C be three non-empty subsets of a set S. Then prove that, $(A\setminus B)\times C=(A\times C)\setminus(B\times C)$. 3

(b) A relation ρ on the set of real numbers $\mathbb{R}$ is defined as follows: $a\rho b$ if and only if $|a|\leq b$. Show that ρ is transitive but neither reflexive nor symmetric. 3

(c) Define injective and surjective mappings. 1+3
Let $f:A\to B$, $g:B\to C$, $h:B\to C$ be three mappings such that f is surjective and $g\circ f=h\circ f$. Prove that $g=h$.

11. (a) Let $\mathbb{N}$ be the set of all positive integers. Let R be the relation on $\mathbb{N}$ defined by $R=\{(a,b)\in\mathbb{N}\times\mathbb{N}: a-b\leq 0\}$. 3
Prove that R is a partial order relation on $\mathbb{N}$.

(b) If $f:S\to T$ is one-one onto, then prove that $f^{-1}:T\to S$ is one-one onto. 3

(c) Prove that a semigroup is a group if it a quasigroup. 4

12. (a) In a group G, prove that $(a^{-1})^{-1}=a$, where $a\in G$ and hence show that group of even order contains an element of order 2. 3

(b) In an Abelian group G, prove that $(ab)^n=a^nb^n$ for all $a, b\in G$, where n is an integer. 3

(c) Prove that anon-empty subset H of a group G is a subgroup of G if and only if for all $a,b\in H$, $a^{-1}b\in H$. 4

13. (a) If R is ring with unity 1, then show that R has characteristic n if and only if $n\cdot 1=0$. 3

(b) Prove that every finite integral domain is a field. 3

(c) Show that $\mathbb{Q}(\sqrt{2})=\{a+b\sqrt{2}: a, b\in\mathbb{Q}\}$ is a subfield of the field $\mathbb{R}$ of real numbers, where $\mathbb{Q}$ is the set of rational numbers. 4

GROUP-C

Answer any *one* question from the following $5\times 1 = 5$

14. If A be a skew-symmetric matrix and $(I+A)$ be a non-singular matrix, then show that $B=(I-A)(I+A)^{-1}$ is orthogonal, where I is the identity matrix of the same size as A. 5

15. Compute the inverse of the matrix A by using row operations, where, 5

$$A=\begin{bmatrix}3 & 1 & 1\\ 4 & 2 & -1\\ 7 & 3 & 1\end{bmatrix}.$$

16. Find the non-singular matrices P and Q such that PAQ is in the normal form and hence find the rank of the matrix 4+1

$$A=\begin{bmatrix}1 & 1 & 1\\ 1 & -1 & -1\\ 3 & 1 & 1\end{bmatrix}.$$

17. Apply Laplace's method along second and third rows to prove that 5

$$\begin{vmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{vmatrix} = (a^2+b^2+c^2+d^2)^2.$$

18. For a square matrix A of order n, prove that $A(\operatorname{adj} A) = (\operatorname{adj} A)A = (\det A)I_n$, where I_n denotes the identity matrix of order n. 5

19. Reduce the quadratic form $2x^2 - 4yz + 6zx + y^2 + z^2$ into its normal form and obtain the rank, signature and nature of the form. 5

GROUP-D

Answer any *one* question from the following $10\times1=10$

20. (a) A company has a manufacturing unit producing two types of products, product A and B. At a time only one type of production is possible. It requires 35 minutes to produce one unit of product A and 25 minutes to produce one unit of product B. The raw material required is 1.4 kgs for one unit of product A and 2.0 kgs for one unit of product B. The factory can run 35 hours per week and the raw material available is 140 kg per week. The profit for the products are Rs. 180 and Rs. 230 per unit of product A and product B respectively. Formulate an LPP to maximize the profit. 5

(b) Solve graphically the LPP: 5

Maximize: $z = 2x_1 + 3x_2$,

Subject to $4x_1 + 3x_2 \leq 12$

$4x_1 + x_2 \leq 8$

$x_1 - x_2 \geq -3$, $x_1, x_2 \geq 0$.

21. (a) Verify whether $x_1 = 1, x_2 = 2, x_3 = 1$ is a feasible solution of the system of equations: 1+4

$$2x_1 + 3x_2 + 5x_3 = 13$$

$$3x_1 - x_2 + 3x_3 = 4$$

Reduce it to a basic feasible solution, if possible.

(b) Show that (2, 0, 1) is a solution of the system of equations 1+2+2

$$2x_1 + 3x_2 + 4x_3 = 8$$

$$x_1 + 3x_2 + 2x_3 = 4$$

Justify whether this is a basic solution. Find another solution of the system which is a basic solution, state basic and non-basic variables.

GROUP-E

Section-I

Answer any *one* question from the following $5\times1=5$

22. Reduce the equation $5x^2 + 4xy + 8y^2 - 14x - 20y - 19 = 0$ to its canonical form and hence identify the conic represented by it. 5

23. Show that the product of the perpendicular distances from the point (x_1, y_1) to the lines represented by $ax^2 + 2hxy + by^2 = 0$ is $\dfrac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a-b)^2 + 4h^2}}$. 5

24. Show that the triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my = 1$ is right angled if $(a+b)(al^2 + 2hlm + bm^2) = 0$. 5

25. If the straight lines $r\cos(\theta - \alpha_i) = p_i$, $i = 1, 2, 3$ are concurrent, prove that $p_1 \sin(\alpha_2 - \alpha_3) + p_2 \sin(\alpha_3 - \alpha_1) + p_3 \sin(\alpha_1 - \alpha_2) = 0$. 5

26. Show that the equation of a circle passing through the pole can be written in the form $r = A\cos\theta + B\sin\theta$, where A, B are constants. 5

Section-II

Answer any *one* question from the following $5\times 1 = 5$

27. Show that the straight lines whose direction cosines are given by $al + bm + cn = 0$ and $fmn + gnl + hlm = 0$ are parallel if $\sqrt{af} \pm \sqrt{bg} \pm \sqrt{ch} = 0$. 5

28. Find the mirror image of the point $(2, -1, 3)$ about the plane $3x - 2y + 7z = 0$. 5

29. Find the equation of the plane passing through the point $(2, 5, -8)$ and perpendicular to each of the planes $2x - 3y + 4z + 1 = 0$ and $4x + y - 2z + 6 = 0$. 5

30. A variable plane with constant distance p from the origin cuts the coordinate axes at A, B, C. Three planes are drawn through the points A, B, C parallel to the coordinate planes. Show that the locus of the points of intersection is given by $\dfrac{1}{x^2} + \dfrac{1}{y^2} + \dfrac{1}{z^2} = \dfrac{1}{p^2}$. 5

31. Find the equation of the straight line passing through the origin and interest both the lines $3x + 2y + 4z - 5 = 0 = 2x - 3y + 4z + 1$ and $2x - 4y + z + 6 = 0 = 3x - 4y + z - 3$. 5

N.B.: *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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