WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 1st Semester Examination, 2018

MTMHGEC01T/MTMGCOR01T-MATHEMATICS (GE1/DSC1)

DIFFERENTIAL CALCULUS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any *five* questions from the following: $2\times 5 = 10$

(a) A function $f(x)$ is defined as follows: 2

$$f(x)=|x-2|+1$$

Examine whether $f'(2)$ exists.

(b) Examine whether $f(x,y)=x^{-1/3}y^{4/3}\cos\left(\frac{y}{x}\right)$ is a homogeneous function of x and y. If so, find its degree. 2

(c) Find the value of $\frac{d^n}{dx^n}\{\sin(ax+b)\}$ 2

(d) Is Rolle's theorem applicable to the function $|x|$ in the interval [–1, 1]? Justify your answer. 2

(e) Find the radius of curvature at the origin for the curve $x^3+y^3-2x^2+6y=0$. 2

(f) Find the asymptotes parallel to co-ordinate axes of the curve $(x^2+y^2)x-ay^2=0$. 2

(g) If $e^{a\sin^{-1}x}=a_0+a_1x+a_2x^2+\ldots\ldots$, then find the value of a_2. 2

(h) Evaluate: $\lim\limits_{x\to 0}(\cos x)^{\cot x}$ 2

2. (a) If $\lim\limits_{x\to a}f(x)$ and $\lim\limits_{x\to a}g(x)$ exists finitely for two functions f and g, then prove that $\lim\limits_{x\to a}\{f(x)+g(x)\}=\lim\limits_{x\to a}f(x)+\lim\limits_{x\to a}g(x)$ 3

(b) Using ε-s definition (Cauchy's definition) show that the function f defined by, 3

$$f(x) \begin{array}{l} = x^2, \quad x \text{ is rational} \\ = -x^2, \quad x \text{ is irrational} \end{array}$$

is continuous at 0.

(c) Find the co-ordinates of the points on the curve $y = x^2 - 8x + 5$ at which the tangents pass through the origin. 2

3. (a) If $f(x) = \begin{cases} x+1 \;, & \text{when } x \le 1 \\ 3 - ax^2 \;, & \text{when } x > 1 \end{cases}$ 3

then find the value of a for which f is continuous at $x = 1$.

(b) Find the Taylor series expansion of $f(x) = \sin x$. 5

4. (a) If $u(x,y) = \tan^{-1}\left(\dfrac{x^3 + y^3}{x - y}\right)$, $x \ne y$, apply Euler's theorem to find the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ and hence show that $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u$ 5

(Assume $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial^2 u}{\partial y \partial x}$)

(b) If $y = \dfrac{x}{x+1}$, find y_n (where y_n is the n-th differential coefficient of y w.r.t x) and hence find $y_7(0)$. 3

5. (a) If $f(x,y) = \begin{cases} xy\dfrac{x^2 - y^2}{x^2 + y^2} \;, & (x,y) \ne (0,0) \\ 0 \;, & (x,y) = (0,0) \end{cases}$ 4

Show that $f_{xy}(0,0) \ne f_{yx}(0,0)$.

(b) Find the asymptotes of the cubic $y^3 + x^2 y + 2xy^2 - y + 1 = 0$ 4

6. (a) State and prove Cauchy's Mean Value Theorem. 5

(b) If $\lim\limits_{x \to 0} \dfrac{\sin 2x + a\sin x}{x^3}$ is finite, find a and the value of the limit. 3

7. (a) Find the radius of curvature at any point (r, θ) for the curve $r = a(1 - \cos\theta)$. Hence show if ρ_1 and ρ_2 be the radii of curvature at the extremities of any chord of this cardioid which pass through the pole; then prove that $\rho_1^2 + \rho_2^2 = \dfrac{16a^2}{9}$ 5

(b) Show that the normal to the curve $3y = 6x - 5x^3$ drawn at the point $\left(1, \frac{1}{3}\right)$ passes through the origin. 3

8. (a) If $H = f(y-z,\ z-x,\ x-y)$, then prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$ 3

(b) Verify Rolle's theorem for the function $f(x) = x^2 + \cos x$ on the interval $[-\frac{\pi}{4}, \frac{\pi}{4}]$. 2

(c) If $V = x\sin^{-1}\left(\frac{y}{x}\right) + y\tan^{-1}\left(\frac{x}{y}\right)$, find the value of $x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y}$ at (1, 1) 3

9. (a) Show that at any point of the curve $by^2 = (x+a)^3$, the subnormal varies as the square of the subtangent. 4

(b) Prove that of all the rectangular parallelopiped of the same volume, the cube has the least surface area. 4

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