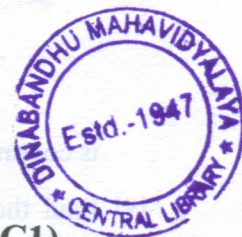




WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 1st Semester Examination, 2018

MTMHGEC01T/MTMGCOR01T-MATHEMATICS (GE1/DSC1)



DIFFERENTIAL CALCULUS

Time Allotted: 2 Hours

Full Marks: 50

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.**All symbols are of usual significance.*

Answer Question No. 1 and any five from the rest

1. Answer any *five* questions from the following: 2×5 = 10
- (a) A function $f(x)$ is defined as follows: 2

$$f(x) = |x - 2| + 1$$
 Examine whether $f'(2)$ exists.
- (b) Examine whether $f(x, y) = x^{-1/3}y^{4/3} \cos\left(\frac{y}{x}\right)$ is a homogeneous function of x and y . If so, find its degree. 2
- (c) Find the value of $\frac{d^n}{dx^n} \{\sin(ax + b)\}$ 2
- (d) Is Rolle's theorem applicable to the function $|x|$ in the interval $[-1, 1]$? Justify your answer. 2
- (e) Find the radius of curvature at the origin for the curve $x^3 + y^3 - 2x^2 + 6y = 0$. 2
- (f) Find the asymptotes parallel to co-ordinate axes of the curve $(x^2 + y^2)x - ay^2 = 0$. 2
- (g) If $e^{a \sin^{-1} x} = a_0 + a_1x + a_2x^2 + \dots$, then find the value of a_2 . 2
- (h) Evaluate: $\lim_{x \rightarrow 0} (\cos x)^{\cot x}$ 2
2. (a) If $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exists finitely for two functions f and g , then prove 3
 that $\lim_{x \rightarrow a} \{f(x) + g(x)\} = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$

- (b) Using ε - δ definition (Cauchy's definition) show that the function f defined by, 3

$$f(x) = \begin{cases} x^2, & x \text{ is rational} \\ -x^2, & x \text{ is irrational} \end{cases}$$

is continuous at 0.

- (c) Find the co-ordinates of the points on the curve $y = x^2 - 8x + 5$ at which the tangents pass through the origin. 2

3. (a) If $f(x) = \begin{cases} x+1, & \text{when } x \leq 1 \\ 3-ax^2, & \text{when } x > 1 \end{cases}$ 3

then find the value of a for which f is continuous at $x = 1$.

- (b) Find the Taylor series expansion of $f(x) = \sin x$. 5

4. (a) If $u(x, y) = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, $x \neq y$, apply Euler's theorem to find the value of 5

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} \text{ and hence show that } x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4 \sin^2 u) \sin 2u$$

$$\left(\text{Assume } \frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}\right)$$

- (b) If $y = \frac{x}{x+1}$, find y_n (where y_n is the n -th differential coefficient of y w.r.t x) 3
and hence find $y_7(0)$.

5. (a) If $f(x, y) = \begin{cases} xy \frac{x^2 - y^2}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ 4

Show that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$.

- (b) Find the asymptotes of the cubic $y^3 + x^2y + 2xy^2 - y + 1 = 0$ 4

6. (a) State and prove Cauchy's Mean Value Theorem. 5

- (b) If $\lim_{x \rightarrow 0} \frac{\sin 2x + a \sin x}{x^3}$ is finite, find a and the value of the limit. 3

7. (a) Find the radius of curvature at any point (r, θ) for the curve $r = a(1 - \cos \theta)$. 5
Hence show if ρ_1 and ρ_2 be the radii of curvature at the extremities of any chord of this cardioid which pass through the pole; then prove that $\rho_1^2 + \rho_2^2 = \frac{16a^2}{9}$

- (b) Show that the normal to the curve $3y = 6x - 5x^3$ drawn at the point $\left(1, \frac{1}{3}\right)$ passes through the origin. 3
8. (a) If $H = f(y - z, z - x, x - y)$, then prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$ 3
- (b) Verify Rolle's theorem for the function $f(x) = x^2 + \cos x$ on the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$. 2
- (c) If $V = x \sin^{-1}\left(\frac{y}{x}\right) + y \tan^{-1}\left(\frac{x}{y}\right)$, find the value of $x \frac{\partial V}{\partial x} + y \frac{\partial V}{\partial y}$ at $(1, 1)$ 3
9. (a) Show that at any point of the curve $by^2 = (x + a)^3$, the subnormal varies as the square of the subtangent. 4
- (b) Prove that of all the rectangular parallelopiped of the same volume, the cube has the least surface area. 4

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