लक्ष्यं विश्वमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2018

MTMACOR01T-MATHEMATICS (CC1)

CALCULUS, GEOMETRY AND ORDINARY DIFFERENTIAL EQUATION

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any five from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Prove that the function $f(x) = A\cos mx + B\sin mx$ satisfies the differential equation $f''(x)+m^2 f(x)=0$.

(b) Find the value of $\lim\limits_{x\to 0}\left[\dfrac{1}{e^x-1}-\dfrac{1}{x}\right]$

(c) From the following parametric equations form an equation in x and y:

$$x=4\sin\left(\frac{t}{4}\right),\ y=1-2\cos^2\left(\frac{t}{4}\right)$$

(d) Write the equation $xy=1$ in terms of a rotated rectangular $x'y'$-system if the angle of rotation from the x-axis to the $x'-$axis is $45°$.

(e) Find the nature of the curve $x^2-y^2+4x+10y=5$

(f) Find the general solution of $3e^x\tan y\,dx+(1-e^x)\sec^2 y\,dy=0$

(g) Find the singular solutions of $\left(\dfrac{dy}{dx}\right)^2+y^2=1$

(h) Test whether the equation $(1+e^{x/y})dx+e^{x/y}\left(1-\dfrac{x}{y}\right)dy=0$ is exact or not.

2. (a) Prove that the asymptotes of the curve $(x^2-4y^2)(x^2-9y^2)+5x^2y-5xy^2-30y^3+xy+7y^2-1=0$ cut the curve in eight points which lie on a circle of unit radius. 5

(b) Show that the envelope of the family of straight lines given by the normal equation : $x\cos C+y\sin C-p=0$ (where C is the parameter) is the circle with radius p and centered at the origin. 3

3. (a) If $y=x^2\cos x$ then prove that 4

$$\frac{d^{n+1}y}{dx^{n+1}}=(n^2+n-x^2)\sin\left(x+\frac{n\pi}{2}\right)+2x(n+1)\cos\left(x+\frac{n\pi}{2}\right),$$

where n is a non-negative integer.

(b) If $\lim\limits_{x\to 0}\dfrac{\sin 2x + a\sin x}{x^3}$ be finite, find the value of a and the limit. 4

4. (a) If $I_{m,n}=\int_0^{\pi/2}\cos^m x\sin nx\,dx$, then prove that $I_{m,n}=\dfrac{1}{m+n}+\dfrac{m}{m+n}I_{m-1,\,n-1}$; 2+2

m, n being positive integers. Hence deduce that

$$I_{m,n}=\frac{1}{2^{m+1}}\left[2+\frac{2^2}{2}+\frac{2^3}{3}+\cdots+\frac{2^m}{m}\right]$$

(b) Find the length of the loop of the curve $x=t^2$, $y=t-\dfrac{t^3}{3}$ 4

5. (a) Show that the area of the surface of the solid generated by revolution the asteroid $x=a\cos^3 t$, $y=a\sin^3 t$ about the axis of x is $\dfrac{12}{5}\pi a^2$ 4

(b) Describe the graph of the ellipse $(x+3)^2+4(y-5)^2=16$ 3

(c) What does the reflexion property of parabola mean? 1

6. (a) Discuss the nature of the conic $x^2+4xy+y^2-2x+2y+a=0$ for different values of 'a'. 4

(b) The latus rectum of a conic is 6 and its eccentricity is $\dfrac{1}{2}$. Find the length of the focal chord making an angle of 45° with the major axis. 4

7. (a) The plane $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$ meets the coordinate axes at A, B, C. Find the equation of the cone generated by the straight lines drawn from O to meet the circle ABC. 4

(b) If a plane passing through a fixed point (α,β,γ) meets the axes at A, B, C respectively, show that the locus of the centre of the sphere passing through the origin and the points A, B, C is $\dfrac{\alpha}{x}+\dfrac{\beta}{y}+\dfrac{\gamma}{z}=2$ 4

8. (a) Solve: $x\,dy-y\,dx=(x^2+y^2)^{1/2}dx$ 4

(b) Solve: $(x^2y-2xy^2)dx-(x^3-3x^2y)dy=0$ 4

9. (a) Find the solution of 4

$$\frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{1}{(1+x^2)^2}$$ under the condition $y=0$ when $x=1$.

(b) Solve: $2x^2\left(\dfrac{dy}{dx}\right)=xy+y^2$ 4

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