लक्ष्यं विश्वमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours PART-III Examinations, 2017

MATHEMATICS-HONOURS

PAPER-MTMA-VII

Time Allotted: 4 Hours — Full Marks: 100

The figures in the margin indicate full marks. Candidates should answer in their own words and adhere to the word limit as practicable. All symbols are of usual significance.

Group-A
(VECTOR ANALYSIS-II)

Answer any *one* question from the following. $10\times1 = 10$

1. (a) Find the constants a, b, c so that the vector $\vec{V}=(x+2y+az)\vec{i}+(bx-3y-z)\vec{j}+(4x+cy+2z)\vec{k}$ is irrotational and then assuming $\vec{V}=\vec{\nabla}\phi$, obtain ϕ. 5

   (b) Evaluate $\oint\limits_{\Gamma}[(\cos x\sin y-xy)dx+\sin x\cos y\ dy]$ by using Green's theorem where Γ is the circle $x^2+y^2=1$ described in the positive sense. 5

2. (a) Prove that $\int\limits_{V}\vec{\nabla}\phi\, dv=\int\limits_{s}\phi\,\hat{n}\, ds$. In particular, show that $\int\limits_{s}\hat{n}\, ds=\vec{0}$. 5

   (b) State Gauss divergence theorem. Use the theorem to show that $\iint_S\vec{r}\,.d\vec{s}=3V$, where V is the volume enclosed by the surface S and $\vec{r}$ has its usual meaning. 5

Group-B
(ANALYTICAL STATICS)

Answer any *five* questions from the following. $7\times5 = 35$

3. A small bead P can slide on a smooth elliptic wire. It is attracted towards the foci S and H by forces proportional to $(SP)^m$ and $(HP)^n$ respectively. Find the position of equilibrium.

4. Define Poinsots central axis of a system of forces acting on a body and show that the central axis is unique.

5. A solid frustum of paraboloid of revolution of height h and latus rectum $4a$, rests with its vertex on the vertex of another paraboloid of revolution whose latus rectum is $4b$. Show the equilibrium is stable if $h < \dfrac{3ab}{a+b}$.

6. A telegraph wire is made of a given material, and such a length l is stretched between two posts, distant d apart and of same length, as will produce the least possible tension at the posts. Show that $l = \dfrac{d}{\lambda}\sinh\lambda$, where λ is given by the equation $\lambda\tanh\lambda = 1$.

7. State and prove the 'principle of virtual work' for any system of coplanar forces acting on a rigid body. If the sum of the virtual works of a system is zero for all displacement, translatory as well as rotatory, then show that the forces are in equilibrium.

8. Two forces act, one along the line $y=0,\ z=0$ and the other along the line $x=0,\ z=c$. As the forces vary, show that the surface generated by the axis of their equivalent wrench is $(x^2+y^2)z = cy^2$.

9. A sphere of weight W and radius r lies within a fixed spherical shell of radius R, and a particle of weight w is attached to its highest point. Show that the equilibrium is stable if

$$W \geq \frac{R-2r}{r}w.$$

10. Two uniform similar rods of same material PQ and QT of lengths $2l$ and $2L$ respectively are rigidly united at Q and suspended freely from P. If they rest inclined at an angle α and β respectively to the vertical, prove that $\left(l^2+2lL\right)\sin\alpha = L^2\sin\beta$.

11. Explain the astatic equilibrium of a system of coplanar forces acting at different points of a body and obtain the astatic centre.

Group-C
(RIGID DYNAMICS)

Answer any *two* questions from the following. $15\times2=30$

12. (a) Explain what is meant by equimomental bodies. Show that a uniform plane triangular lamina is equimomental with a system of three particles placed at the middle points of the sides, each equal to one third the mass of the triangle. 8

(b) A plank of mass M is initially at rest along a straight line of greatest slope of a smooth plane inclined at an angle α to the horizon, and a man, of mass M' starting from the upper end walks down the plank so that it does not move. Show that he gets to other end in time $\sqrt{\dfrac{2M'a}{(M+M')g\sin\alpha}}$, where a is the length of the plank and g is the acceleration due to gravity. 7

13. (a) State D'Alembert's principle and deduce the equation of motion of the centre of inertia of a rigid body and the equation of motion relative to the centre of inertia. 8

(b) A right cone of angle 2α, can turn freely about an axis passing through the centre of base and perpendicular to its axis. If the cone starts from rest with its axis horizontal, show that when the axis is vertical, the thrust on the fixed axis is to the weight of the cone as $\left(1+\frac{1}{2}\cos^2\alpha\right):\left(1-\frac{1}{3}\cos^2\alpha\right)$. 7

14. (a) Show that the resultant kinetic energy of a rigid body moving in two dimensions under finite forces is equal to the sum of two kinetic energies, one due to translation and the other due to rotation. 8

(b) A rough uniform rod, of length $2a$, is placed on a rough table at right angles to its edge. If the centre of gravity of the rod be initially at a distance b beyond the edge, then show that the rod will begin to slide when it has turned through an angle given by $\tan\theta=\dfrac{\mu a^2}{a^2+9b^2}$. 7

Group-D
(HYDROSTATICS)

Answer any *two* questions taking *one* question from each section.

SECTION-I

15. (a) Show that in a homogeneous fluid at rest under gravity, the difference of the pressures between any two points is proportional to the differences of their depths. 8

(b) A vertical circular cylinder of height $2h$ and radius r closed at the top, is just filled by equal volumes of two liquids of density ρ and σ. Show that if the axis be gradually inclined to the vertical, the pressure at the lowest point of the base will never exceed 7

$$g(\rho+\sigma)(r^2+h^2)^{\frac{1}{2}}.$$

16. (a) Prove that the necessary and sufficient condition for equilibrium of a fluid under the action of a force whose components are X, Y, Z along the coordinate axes is that 8

$$X\left(\frac{\partial Y}{\partial z}-\frac{\partial Z}{\partial y}\right)+Y\left(\frac{\partial Z}{\partial x}-\frac{\partial X}{\partial z}\right)+Z\left(\frac{\partial X}{\partial y}-\frac{\partial Y}{\partial x}\right)=0$$

(b) A quadrant of a circle is immersed in a liquid with a bounding radius in the surface. Find the position of its centre of pressure. 7

SECTION-II

17. (a) A hemispherical surface of radius a is immersed in a liquid of density ρ with its centre at a depth h and its base inclined at an angle θ to the horizontal. Find the resultant thrust on the curved surface. 5

(b) A liquid fills the lower half of a circular tube of radius a in a vertical plane. If the tube is now rotated about the vertical diameter with uniform angular velocity ω such that the liquid is about to separate in two parts, show that $\omega=\sqrt{\frac{2g}{a}}$. 5

18. (a) A cone of density ρ whose height is h and the radius of whose base is a floating with its axis vertical and vertex upwards in liquid of density σ. Prove that the equilibrium is stable if $\rho<\sigma(1-\cos^6\alpha)$. 5

(b) Prove that if the temperature in the atmosphere falls uniformly with the height ascended, the height of a station above the sea level is given by $z=a\left\{1-\left(\frac{h}{h_0}\right)^m\right\}$, where h, h_0, are the reading of the barometer at station and sea level respectively and a, m are constants. 5

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