लक्ष्यं चिन्मयानन्

# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours PART-III Examinations, 2017

## MATHEMATICS-HONOURS

## PAPER-MTMA-VI

Time Allotted: 4 Hours | Full Marks: 100

*The figures in the margin indicate full marks. Candidates should answer in their own words and adhere to the word limit as practicable. All symbols are of usual significance.*

### Group- A

**Answer any *two* questions from Question No. 1 to 3 and any *one* from Question No. 4 and 5.**

1. Answer any *three* questions from the following: $5\times3 = 15$

(a) Give the axiomatic definition of probability. Deduce the classical definition from the axioms. 2+1+2
Give the frequency interpretation of the axioms of probability.

(b) Define conditional probability. 1+4
If $A$, $B$ and $C$ are any three events, then prove that
$P(A+B\mid C)=P(A\mid C)+P(B\mid C)-P(AB\mid C)$
where $P(C)\neq 0$.

(c) From the numbers 1, 2, …., $(2n+1)$ three are chosen at random. Prove that the probability that these are in arithmetical progression is $\dfrac{3n}{(4n^2-1)}$. 5

(d) From an urn containing 3 white and 5 black balls, 4 balls are transferred into an empty urn. From this urn a ball is drawn and is found to be white. What is the probability that out of the four balls transferred 3 are white and 1 is black? 5

(e) (i) Show that if two events $A$ and $B$ be independent then $\overline{A}$ and $\overline{B}$ are also independent. 2

(ii) $A_1, A_2, \ldots, A_n$ are $n$ mutually independent events. Prove that the probability of occurrence of at least one of the events is $1-(1-p_1)(1-p_2)\ldots(1-p_n)$, where $P(A_i)=p_i$ and $P(\overline{A}_i)=1-p_i$, $i=1,2,\ldots n$. 3

2. Answer any *three* questions from the following: $5\times 3 = 15$

(a) State the central limit theorem for the case of equal components and show that the theorem implies the law of large numbers. 2+3

(b) Consider the distribution function of $X$ given by 1+1+1.5+1.5

$$F(x)=\begin{cases} 0, & \text{for } x<0 \\ 1-\dfrac{1}{4}, & \text{for } x\ge 0 \end{cases}$$

Determine $P(X=0)$, $P(X>0)$, $P(1<X\le 2)$ and $P(1.5\le X\le 3)$.

(c) If $X$ be a $\gamma\left(\frac{1}{2}\right)$ variate, then find the probability density function of $\frac{1}{2}X^2$. 5

(d) If $X$ is a standard normal variate, find the mean and variance of $e^x$. 5

(e) $X$ is a Poisson variate with parameter $\mu$. 5

Show that $P(X\le n)=\dfrac{1}{n!}\int_{\mu}^{\infty} e^{-x}x^n dx$, where $n$ is a positive integer.

3. Answer any *three* questions from the following: $5\times 3 = 15$

(a) Show that the mean deviation about the mean of a normal $(m,\sigma)$ distribution is $\sqrt{\dfrac{2}{\pi}}\,\sigma$. 5

(b) If $X$ and $Y$ are two correlated random variables with same standard deviation, show that the correlation coefficient between $X$ and $X+Y$ is $\sqrt{\frac{(1+\rho)}{2}}$, where $\rho$ is the correlation coefficient between $X$ and $Y$. 5

(c) Two numbers are independently chosen at random between 0 and 1. Show that the probability that their product is less than a constant $k(0<k<1)$ is $k(1-\log k)$. 5

(d) Find the moment generating function of a random variable $X$ uniformly distributed in $(-a, a)$ and hence find $E(X^n)$, $n$ a positive integer. 5

(e) A random variable $X$ has a density function $f(x)$ given by 5

$$f(x)=e^{-x},\ x\geq 0.$$
$$=0,\ \text{elswhere}.$$

Show that Tchebycheff's inequality gives $P(|X-1|\geq 2)\leq\frac{1}{4}$. Show that the actual probability of this event is $e^{-3}$.

4. (a) Obtain the maximum likelihood estimator of $\sigma^2$ when $\mu$ is known, where $\mu$ and $\sigma$ are mean and standard deviations of a normal population. Show that this estimator is unbiased. 5

(b) For a normal population, show that the sampling distribution of the statistic $\chi^2=\frac{ns^2}{\sigma^2}$ is $\chi^2$ - distribution with $(n-1)$ degrees of freedom, where $n$, $s^2$ and $\sigma^2$ are respectively the sample size, sample variance and population variance. 7

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(c) Define 'critical region' for testing a statistical hypothesis. The random variable $X$ denoting the amount of consumption of a commodity follows the distribution

$$f(x,\theta)=\frac{1}{\theta}\,.e^{-\frac{x}{\theta}},\qquad 0<x<\infty,\ \theta>0.$$

The hypothesis $H_o$: $\theta = 5$ is rejected in favour of $H_1 : \theta = 10$, if 15 units or more chosen randomly be consumed. Obtain the size of the two types of errors and the power of the test.

5. (a) The bivariate probability density function of two random variables $X$ and $Y$ is given by 8

$$f(x,y)=x+y,\ \ 0<x<1,\ \ 0<y<1$$
$$=0 \qquad ,\text{elsewhere}.$$

Calculate the means, standard deviations of $X$ and $Y$ and also the correlation coefficient between $X$ and $Y$. Find the equations of two regression lines.

(b) For two random variables $X$ and $Y$ with the same mean, the two regression lines are $y = ax + b$ and $x = \alpha y + \beta$. 5

Show that $\frac{b}{\beta}=\frac{1-a}{1-\alpha}$. Find also the common mean.

(c) The heights of 10 males of a normal population are found to be 70, 67, 62, 67, 61, 68, 70, 64, 65, 66 inches. Is it reasonable to believe that the average height is greater than 64 inches? Test at 5% significance level, assuming that for 9 degrees of freedom $P(t > 1.83) = 0.05$. 7

## Group-B

## Section- I

**[Marks- 30]**

**Answer any *three* questions from the following.** $10\times3=30$

6. (a) (i) Show that $\Delta^2 \cos 2x = 4\cos 2x$ where interval of differencing is $\frac{\pi}{2}$. 5

   (ii) Prove that $\Delta \tan^{-1}\left(\frac{x-1}{x}\right) = \cot^{-1}\left(2x^2\right)$ for unit interval of differencing.

   (b) Define the terms absolute and relative errors. What are inherent errors? How do they arise? What is chopping? 1+1+1+1+1

7. (a) Explain the Newton-Raphson method for computing a simple root of an equation $f(x) = 0$. When does the method fail? 4+1

   (b) Obtain Lagrange's interpolation formula (without error term). 5

8. (a) Describe Gauss-Seidel method for numerical solution of a system of linear equations and state the condition of convergence. 4+1

   (b) Find the Simpson's $\frac{1}{3}$ composite rule for numerical integration. State the error term for Simpson's $\frac{1}{3}$ rule, and hence find the degree of precision of this rule. 5

9. (a) Deduce numerical differentiation formula (both 1st and 2nd order) from Newton's forward interpolation formula. 5

(b) Using Picard's method, find a solution of $\frac{dy}{dx} = 1 + xy$ up to the third approximation, when $x_0 = 0,\ y_0 = 0$.

10.(a) Describe the power method to calculate the greatest eigenvalue of a real square matrix of order $n$.

(b) Show that in approximating $e^x$ by the interpolation polynomial using points $1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}$, the remainder is of the form $(x-1)\left(x-\frac{1}{2}\right)\ldots\left(x-\frac{1}{n}\right)\frac{e^{\theta}}{(n+1)!}$ where $\min\left\{x, \frac{1}{n}\right\} < \theta < \max\{x, 1\}$.

## Section- II

**[Marks- 20]** $10\times2=$

**Answer any *two* questions from the following.**

11.(a) Write a ForTran or C program to add the first 10 terms of the series

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots..$$

(b) Write a ForTran or C program to determine whether a number is prime or not.

12.(a) Write a ForTran or C program to calculate and print the

(i) mean

(ii) variance and

(iii) standard deviation of the following set of numbers:

7.5, 2.6, 3.2, 8.9, 7.2, 8.1, 5.6

(b) Write a short note on if-else statements giving a suitable example. 3

13.(a) Write the full forms of ALU, ROM, CPU, CNF. What is a flowchart? 2+1

(b) Draw a flowchart to determine the smallest of three numbers. 2

(c) Write a ForTran or C program to find the sum of two $m \times n$ matrices. 5