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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours PART-I Examinations, 2016

MATHEMATICS-HONOURS

PAPER-MTMA-I

Time Allotted: 4 Hours Full Marks: 100

The figures in the margin indicate full marks. Candidates should answer in their own words and adhere to the word limit as practicable. All symbols are of usual significance.

Group-A

Answer any *five* questions from the following $5\times5 = 25$

1. (a) If two integers 'a' and 'b' be prime to each other, then prove that $(a + b)$ is prime to ab. 3

   (b) If G.C.D. $(a, b) = 1 =$ G.C.D.(a, c), then show that G.C.D. $(a, b, c) = 1$. 2

2. (a) If p be a prime and be not a divisor of a, then show that $a^{p-1}\equiv 1 (\text{mod}\ p)$. 3

   (b) Show that the number of prime integers is infinite. 2

3. (a) Use congruence to show that $2^{5n+3}+5^{2n+3}$ is divisible by 7. 3

   (b) Show that the square of an odd integer is of the form $8k + 1$. 2

4. Show that the general value of $(1+i\tan\alpha)^{-i}$ is 5

$$e^{\alpha+2m\pi}\{\cos(\log\cos\alpha)+i\sin(\log\sin\alpha)\}.$$

5. If $\tan(\theta+i\phi)=\sin(\alpha+i\beta)$, then show that $\cot\beta\sinh 2\phi=\cot\alpha\sin 2\theta$. 5

6. (a) If $x+\frac{1}{x}=2\cos\frac{\pi}{7}$, then show that $x^7+\frac{1}{x^7}=-2$. 3

(b) Express $\frac{-1+i\sqrt{3}}{1+i}$ in polar form. 2

7. Solve the equation $x^5-5x^4+9x^3-9x^2+5x-1=0$. 5

8. (a) Remove the second term of the equation $x^3+6x^2+12x-19=0$ and hence solve the given equation. 1+2

(b) If α be a root of the Cubic equation $x^3-3x+1=0$, then show that the other roots are (α^2-2) and $(2-\alpha-\alpha^2)$. 2

9. (a) If α,β,γ be the roots of the equation $x^3-9x+9=0$, then show that $(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)=\pm 27$. 3

(b) Apply Descartes' rule of signs to show that the equation $x^4+2x^2-7x-5=0$ has two real roots and two non-real roots. 2

Group-B

Answer any *two* questions from the following $10\times 2=20$

10.(a) If X, Y, A be three non-empty sets such that $A\cap X=A\cap Y$ and $A\cup X=A\cup Y$, then prove that $X=Y$. 3

(b) Define an equivalence class of a relation R on a set A. Prove that any two equivalence classes are either disjoint or equal. 1+3 = 4

(c) Prove that a finite integral domain is a field. 3

11.(a) Prove that intersection of any two equivalence relations on a set is an equivalence relation on that set. 2

(b) Let $A=\{a, b, c\}$ and Let $f:A\to A$, $g:A\to A$ be given by $f(a) = b$, $f(b) = c$, $f(c) = a$; $g(a) = a$, $g(b) = c$, $g(c) = b$. Evaluate $f\circ g$ and $g\circ f$ and show that $f\circ g \neq g\circ f$. 1+1+1 = 3

(c) Let $(G, *)$ be a semi-group and for $a,b \in G$ each of the equations $a * x = b$ and $y * a = b$ has a solution in G. Then show that $(G, *)$ is a group. 5

12.(a) Show that the set of integers modulo 6 form a ring with respect to addition and multiplication modulo 6. Is this an integral domain? Justify your answer. 4+1 = 5

(b) If a be a fixed element of the group G, then show that the set $N(a)=\{x\in G \mid xa=ax\}$ is a subgroup of G. 5

13.(a) If $(R,+,\cdot)$ be a ring such that $a^2=a$, for all $a\in R$, then prove that $(R,+,\cdot)$ is a commutative ring. 5

(b) If $a,b,c\in F$ a field and $a\neq 0$, then show that the equation $ax+b=c$ has a unique solution in F. 5

Group-C

Answer any *three* questions from the following 5×3 = 15

14. Show that $\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix} = 2s^3(s-a)(s-b)(s-c)$, 5

where $2s=a+b+c$.

15. Find the value(s) of k for which the system of equations 5

$$x+y-z=1,$$
$$2x+3y+kz=3,$$
$$x+ky+3z=2$$

has (i) no solution, (ii) unique solution, (iii) more than one solution.

16. If A and B be two matrices, such that $AB = A$ and $BA = B$, then show that A and B are idempotent. 5

17. If the sum of the elements in each row of a real non-singular matrix be $k(\neq 0)$, then show that the sum of the elements in each row of the inverse matrix is k^{-1}. 5

18. Let A and B be real orthogonal matrices of the same order such that $|A| + |B| = 0$. Show that $(A + B)$ is a Singular matrix. 5

19. Reduce the real quadratic form $2x^2+2y^2+5z^2-4xy-2zx+2yz$ to its normal form. Then show that the quadratic form is positive semi-definite. 5

Group-D

Answer any *one* question from the following $10\times1 = 10$

20.(a) A factory is engaged in manufacturing three products A, B and C which involve lathe work, grinding and assembling. The cutting, grinding and assembling times required for one unit of A are 2, 1 and 1 hours respectively. Similarly they are 3, 1, 3 hours for one unit of B and 1, 3, 1 hours for one unit of C. The profits on A, B and C are Rs. 2, Rs. 2 and Rs. 4 per unit respectively. There are available 300 hours of lathe time, 300 hours of grinder time and 240 hours of assembly time. Formulate an LPP for maximum profit. 5

(b) Solve the L.P.P. by graphical method : 5

Maximize $Z = 4x_1 + 3x_2$

Subject to $3x_1 + 4x_2 \le 12,$

$2x_1 + 5x_2 \le 10,$

$x_1 + x_2 \ge 1,$

$x_1, x_2 \ge 0.$

21.(a) Show that $x_1 = 2$, $x_2 = 2$, $x_3 = 3$ is a feasible solution of the system of equations: 1+4 = 5

$$4x_1 + 2x_2 - 3x_3 = 1,$$

$$-6x_1 - 4x_2 + 5x_3 = -1.$$

Reduce the feasible solution to a basic feasible solution.

(b) Two linearly independent equations with three variables are given below: 5

$$3x_1 + x_2 - 6x_3 = 1,$$

$$2x_1 - 3x_2 - 4x_3 = 5.$$

Find, if possible, the basic solution with x_2 as a non-basic variable.

Group-E

Section-I

Answer any *three* questions from the following 5×3 = 15

22. Reduce the equation $3x^2 + 10xy + 3y^2 - 12x - 12y + 4 = 0$ to its canonical form and state the name of the conic represented by it. 5

23. PSP' is a focal chord of the conic $\frac{l}{r} = 1 + e\cos\theta$. Prove that the angle between the tangents at P and P' is $\tan^{-1}\left(\frac{2e\sin\alpha}{1-e^2}\right)$, where α is the angle between the chord and the major axis. 5

24. If the equation $ax^2 + 2hxg + by^2 + 2gx + 2fy + c = 0$ represents two straight lines equidistant from the origin, then show that $f^4 - g^4 = c(bf^2 - ag^2)$. 5

25. The straight line $ax + by + c = 0$ bisects an angle between a pair of straight lines of which one is $lx + my + n = 0$. Show that the other line of the pair is $(lx + my + n)(a^2 + b^2) - 2(al + bm)(ax + by + c) = 0$. 5

26. Find the polar equation of the chord joining two points on the conic $\frac{l}{r} = 1 - e\cos(\theta - \gamma)$ with $\alpha - \beta$ and $\alpha + \beta$ as their vectorial angles. Hence find the equation of the tangent to the conic at $\theta = \alpha$. 5

Section-II

Answer any *three* questions from the following. 5×3 = 15

27. Find the image of the point (4, –2, 3) in the plane $2x - 3y + z = 7$. 5

28. Show that if the straight lines whose direction cosines are given by $al+bm+cn = 0$, $fmn + gnl + hlm = 0$ be parallel, then one of the relations $\sqrt{af} \pm \sqrt{bg} \pm \sqrt{ch} = 0$ is true. 5

29. Find the shortest distance and its equation between the lines $\frac{x}{4} = \frac{y+1}{3} = \frac{z-2}{2}$, and $5x - 2y - 3z + 6 = 0 = x - 3y + 2z - 3$. 5

30. Find the equations of the lines of greatest slope and least slope on the plane $3x - 4y + 5z - 5 = 0$ drawn through the point (1, 2, 2) given that the plane $4x - 5y + 6z - 6 = 0$ is horizontal. 5

31. Show that the surface generated by straight lines $y = 0, z = c$; $x = 0, z = -c$ and the curve $z = 0, xy + c^2 = 0$ is $z^2 - c^2 = xy$. 5