लक्ष्यं विद्यमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours PART-III Examination, 2016

MATHEMATICS-HONOURS

Paper-MTMA-VII

Time Allotted: 4 Hours | Full Marks: 100

The figures in the margin indicate full marks. Candidates should answer in their own words and adhere to the word limit as practicable. Symbols are of usual significance.

Group-A

(Vector Analysis-II)

Answer any *one* question from the following: $10\times1 = 10$

1. (a) Prove that $\iiint_V \frac{dV}{r^2} = \iint_S \frac{\vec{r}.\vec{n}}{r^2} dS$ where S is any closed surface enclosing a volume V. 5

(b) If $\vec{F}=(2x-y+4z)\vec{i}+(x+y-z^2)\vec{j}+(3x-2y+4z^3)\vec{k}$, then evaluate $\int_C \vec{F}.\vec{dr}$ over the circle $x^2+y^2=9,\ z=0$. 5

2. (a) Applying Green's theorem show that the area bounded by a simple closed curve C is given by $\frac{1}{2}\oint_C (xdy - ydx)$. Hence obtain the area of the ellipse $x = a\cos t$, $y = b\sin t$. 5

(b) Verify divergence theorem for the factor function $\vec{F} = 2xz\hat{i} + y^2\hat{j} + yz\hat{k}$ taken over the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. 5

Group-B

(Analytical Statics)

Answer any *five* questions from the following: $7\times5 = 35$

3. A force parallel to the axis of z acts at the point $(a, 0, 0)$ and an equal force perpendicular to the axis of z acts at the point $(-a, 0, 0)$. Show that the central axis of the system lies on the surface $z^2(x^2 + y^2) = (x^2 + y^2 - ax)^2$.

4. A solid body, consisting of a cone and hemisphere on the same base, rests on a rough horizontal table, the hemisphere being in contact with the table; show that the greatest height of the cone, so that the equilibrium may be stable, cannot exceed $\sqrt{3}$ times the radius of the hemisphere.

5. Explain the terms 'force of friction' and the 'angle of friction'.

A uniform rod rests in limiting equilibrium within a rough hollow sphere; if the rod subtends an angle 2α at the centre of the sphere and if λ be the angle of friction, show that the angle of inclination of the rod to the horizon is

$$\tan^{-1}\left[\frac{\sin 2\lambda}{\cos 2\alpha+\cos 2\lambda}\right].$$

6. Two uniform similar rods of same material PQ and QT of lengths $2a$ and $2b$ respectively are rigidly united at Q and suspended freely from P. If they rest inclined at an angle α and β respectively to the vertical, prove by the principle of virtual work that $(a^2+2ab)\sin\alpha=b^2\sin\beta$.

7. What is Static equilibrium? Prove that if a system of coplanar forces acting at different points of a body have a single resultant and if each force be turned in the plane of forces about its point of application through the same angle and same sense then their resultant will always pass through a fixed point.

8. What is the energy test of stability? Establish the energy test of stability for a rigid body with one degree of freedom only, in equilibrium under conservative forces.

9. A body rests in equilibrium on another fixed body, there be enough friction to prevent sliding, the portion of the two bodies in contact are spherical and of radii r and R respectively and the line joining their centers in position of equilibrium is vertical. Show that the equilibrium is stable if $\frac{1}{h} > \frac{1}{r} + \frac{1}{R}$, where h is the height of the C.G. of the body in position of equilibrium above the point of contact.

10. A regular hexagon is composed of six equal heavy rods each of weight w and the rods are freely joined together. Two opposite angles are connected by a string, which is horizontal, one rod being in contact with horizontal plane, at the middle of the opposite rods is placed a weight $\sqrt{3}w$. Show that the tension of the string is $\left(\sqrt{3}+1\right)w$.

11. Find the centre of gravity of a plane lamina of uniform density in the form of a quadrant of an ellipse.

Group-C

(Rigid Dynamics)

Answer any *two* questions from the following: $15\times2 = 30$

12. (a) Prove that moment of inertia of a triangular lamina ABC about a perpendicular to the plane through the vertex A is $\frac{M}{3}(3b^2 + 3c^2 - a^2)$, where a, b, c are length of the sides of the triangle and M is its mass. 8

(b) A solid homogeneous cone, of height h and vertical angle 2α, oscillates about a horizontal axis through its vertex; show that the length of the simple equivalent pendulum is $\frac{h}{3}(4 + \tan^2 \alpha)$. 7

13. (a) Find the moment of momentum of a rigid body moving in two dimension about the origin. 7

(b) A uniform rod of length $2a$ and weight w is turning about its end O and starts turn the position in which it was vertically above O. When it has turned through an angle θ, show that the horizontal and vertical reactions about O are $\frac{3w}{4}\sin\theta(2 - 3\cos\theta)$ and $\frac{w}{4}(1-3\cos\theta)^2$. 8

14. (a) Establish the principle of independence of the motions of translation and rotation of a rigid body. 8

(b) Two equal uniform rods AB and AC are freely joined at A. They are placed on a table so as to be at right angles. The rod AC is struck by a blow at C in a direction perpendicular to AC. Show that the resulting velocities of the middle points of AB and AC are in ratio 2:7. 7

Group-D

(Hydrostatics)

Answer any *two* questions taking *one* question from each section

SECTION-I

15. (a) Explain the term 'perfect fluid'. Show that the pressure at a point in a fluid in equilibrium is the same in every direction. 8

(b) A Cycloidal uniform tube contains equal weight of two liquids occupying lengths a and b. If it be placed with its axis vertical, prove that heights of the free surfaces of the fluids above the vertex of the tube areas $(3a+b)^2 : (3b+a)^2$. 7

16. (a) A quantity of fluid revolves uniformly and without any relative displacement of the particles about a fixed axis z. Determine the nature of the surfaces of equi-pressure. 8

(b) If an area is bounded by two concentric semi-circles with their common bounding diameter in the free surface, prove that the depth of the centre of pressure is $\frac{3\pi(a+b)(a^2+b^2)}{a^2+b^2+ab}$, where a and b are their radii ($a > b$). 7

Symmetry

SECTION-II

17. (a) A cylinder makes vertical oscillation in a liquid contained in another cylinder, the radius of which is n times of the former; show that the length of the axis immersed when it is in a position of rest is $\frac{gT^2n^2}{4\pi^2(n^2-1)}$, where T is the time of a complete oscillation. 5

(b) A body floats partially immersed in a liquid and is free to turn about a fixed point O of the body. Find the necessary and sufficient conditions of equilibrium of the body. 5

18. (a) Find the condition for existence of metacentre of a body and prove the formula $HM = \frac{AK^2}{V}$, with usual notations, for finding the metacentre of the body floating freely in a homogeneous liquid at rest under gravity. 5

(b) Prove that in a fluid at rest under gravity, the pressure is the same at all points in the same horizontal plane. 5