सह्यं विश्वमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours PART-III Examination, 2016

MATHEMATICS-HONOURS

Paper-MTMA-VI

Time Allotted: 4 Hours | Full Marks: 100

The figures in the margin indicate full marks. Candidates should answer in their own words and adhere to the word limit as practicable. Symbols are of usual significance.

Group- A

Answer any *two* questions from Question No. 1 to 3 and any *one* from Question No. 4 and 5

1. Answer any *three* of the following questions: $5\times3=15$

(a) If $\{A_n\}$ be a monotonic sequence of events, then show that 5

$$P\left(\lim_{n\to\infty} A_n\right)=\lim_{n\to\infty} P(A_n).$$

(b) Four dice are thrown. Find the probability that the sum of the numbers is 18. 5

(c) If n objects are distributed among 'a' men and 'b' ($b < a$) women, show that probability that the women get an odd number of objects is 5

$$\frac{\frac{1}{2}\left[(a+b)^n-(a-b)^n\right]}{(a+b)^n}$$

(d) Define conditional probability $P(A/B)$. Show that conditional probabilities satisfy all the three axioms of probability. 5

(e) Find the probability density function $f(x)$ corresponding to the probability distribution function 5

$$F(x)=\begin{cases} 0, & -\infty < x < 0 \\ 1-e^{-x}, & 0 \le x < \infty \end{cases}.$$

2. Answer any *three* questions: 5×3=15

(a) State Tchebycheff's theorem and hence prove Bernoulli's limit theorem. 5

(b) Derive Poisson approximation to the binomial Law in the limiting case. 5

(c) If X is a $\gamma(l)$ variate, find the probability density function of $\sqrt{X}$. 5

(d) Let λ and μ_r denote the mean and r-th moment about the mean respectively of a Poisson distribution. Obtain the following recurrence relation: 5

$$\mu_{r+1} = \lambda\left[r\mu_{r-1} + \frac{d\mu_r}{d\lambda}\right].$$

(e) Find the median and mode, if any, of the distribution having probability density function $f(x) = \lambda e^{-\lambda x}$, $(\lambda > 0,\ x > 0)$. 5

3. Answer any *three* of the following: 5×3=15

(a) The joint probability density function of the random variable x and y is $f(x,y)=\begin{cases} 6(1-x-y), \text{ for } x \ge 0,\ y \le 0,\ x+y<1 \\ 0 \text{ elsewhere.} \end{cases}$ 5

Find the (i) marginal probability density functions,
(ii) the covariance of x and y.

(b) If X_n be a Binomial (n, p) variate, then show that $\frac{(X_n - np)}{\sqrt{npq}}$ is asymptotically Normal (0, 1). 5

(c) State Bernoulli's theorem and the Law of Large numbers. Obtain Bernoulli's Theorem as a particular case of the Law of Large numbers for equal components. 5

(d) A vertical board is ruled with horizontal parallel lines at a constant distance d apart. A needle of length $l(< d)$ is thrown at random on the board. Find the probability that it will intersect one of the lines. 5

(e) Let (X, Y) be a two- dimensional random variable. Prove that $[E(XY)]^2 \le E(X^2)E(Y^2)$. Hence show that $-1 \le \rho \le 1$, where ρ is the correlation coefficient between X and Y. 5

4. (a) Define an unbiased and consistent estimate of a parameter connected with the distribution function of a population. Prove that sample mean is always unbiased and consistent estimate of the population mean. 6

(b) Find the maximum likelihood estimate of the parameter λ of a continuous population having the density function $f(x) = \lambda x^{\lambda-1}$, $(0 < x < 1)$, where $\lambda > 0$. 6

(c) Let $s_1^2, s_2^2, \ldots\ldots, s_k^2$ denote a sample of size k from the population of the unbiased estimate of the population variance for a normal (m, σ) population. Write down the likelihood function for this sample and show that maximum likehood estimate of σ^2 is given by, $\hat{\sigma}^2 = \dfrac{\left(s_1^2 + s_2^2 + \ldots\ldots + s_k^2\right)}{k}$. 8

5. (a) The regression lines for a bivariate sample are given by $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$ and let $s_x^2 = 12$. Calculate the values of $\bar{x}, \bar{y}, s_y$ and r. 8

(b) Write a short note on χ^2 test of goodness of fit of a random sample to a hypothetical distribution. 5

(c) Obtain the best critical region for the population 7

$$f(x, \theta) = \theta e^{-\theta x},\ x \geq 0$$

of the size α for testing $H_o : \theta = \theta_o$ against the alternative $H_1 : \theta = \theta_1$.

Group- B

Answer any *three* questions from Section-I and any *two* from Section-II.

Section- I

[Marks- 30]

6. (a) Define n th order divided difference of a function $f(x)$ based on interpolating points $x_0, x_1, \ldots\ldots, x_n$. If the interpolating points are equally spaced, $x_i = x_o + rh,\ r = 0, 1, 2, \ldots, n,$ then show, in usual symbol, that 5

$$f(x_0, x_1, \ldots., x_n) = \frac{\Delta^n f(x_n)}{n!\, h^n}.$$

(b) Show that $\Delta \log f(x) = \log\left\{1 + \frac{\Delta f(x)}{f(x)}\right\}$. 5

7. (a) If an equation $f(x) = 0$ has a real root, prove that bisection method is unconditionally convergent to the root and that the convergent is linear. 5

(b) Describe the method of false position for finding a real root of an equation $f(x) = 0$ and obtain the corresponding iteration formula. Discuss its advantages and disadvantages in comparison to Newton-Raphson method. 5

8. (a) State the general principle of quadrature formula for evaluating an integral of the form $\int_a^b f(x)dx$, where a, b are finite. Hence or otherwise obtain the trapezoidal rule. Is it a closed type formula? Justify your answer. 5

(b) Explain the method of fixed point iteration for numerical solution of the equation of the form $x = \phi(x)$. Derive the condition of convergence. 5

9. (a) Explain the principle of numerical differentiation. Deduce Lagrange's numerical differentiation formula (without error term). 5

$\phi'(\xi_n)$

(b) What are basic concepts of Hermite interpolation? Prove the uniqueness of Hermite interpolation polynomial. 5

10. (a) Write down the steps to be followed to solve for $X = (x_1, x_2, \ldots, x_n)^T$ from the $n \times n$ system of linear equations $AX = B$ by Gauss-elimination method with partial pivoting, where $A = (a_{ij})_{n \times n}$ is the coefficient matrix and $B = (b_1, b_2, \ldots, b_n)^T$. 5

(b) Solve the equation $\frac{dy}{dx} = y^2 + yx,\ y(1) = 1$ by modified Euler's method to obtain $y(1.2)$ and $y(1.4)$. 5