West Bengal State University

B.A./B.Sc./B.Com. (Honours, Major, General) Examinations, 2015

PART-II

MATHEMATICS – Honours

Paper– IV

Duration : 4 Hours Full Marks : 100

The figures in the margin indicate full marks.

Group - A

Answer any *two* questions. $2 \times 10 = 20$

1. a) Show that the pole of any tangent to the hyperbola $xy = c^2$ with respect to the circle $x^2 + y^2 = a^2$ lies on concentric and similar hyperbola. 5

   b) Define chord of contact of tangents. Find the equation of the pair of tangents from an external point (x_1, y_1) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 2 + 3

2. a) Find the equation of the sphere touching the three coordinate planes. 5

   b) Prove that the conditions that the lines of section of the plane $lx+my+nz=0$ and the cones $ax^2+by^2+cz^2=0$, $fyz+gzx+hxy=0$ may be coincident are $\frac{bn^2+cm^2}{fmn} = \frac{cl^2+an^2}{gnl} = \frac{am^2+bl^2}{hlm}$. 5

3. a) Show that the enveloping cylinder of the conicoid $ax^2+by^2+cz^2=1$ with generators perpendicular to the z-axis meets the plane $z = 0$ in a pair of straight lines. 5

   b) Reduce the equation $x^2-y^2+4yz+4zx-6x-2y-8z+5=0$ to its canonical form and examine the nature of the conic it represents. 5

Group - B

Answer any *one* question. $1 \times 10 = 10$

4. a) Find the eigen values and the corresponding eigenfunction of the eigenvalue problem

   $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + \lambda y = 0$, $(\lambda > 0)$ satisfying the boundary conditions $y'(1)=0$ and $y'(e^{2\pi})=0$. 5

   b) Solve : $2\frac{d^2x}{dt^2} - \frac{dy}{dt} - 4x = 2t$,

   $2\frac{dx}{dt} + 4\frac{dy}{dt} - 3y = 0$. 5

5. a) Find the equation of the integral surface given by the differential equation
$2y(z-3)p+(2x-z)q = y(2x-3)$, which passes through the circle $z = 0$, $x^2 + y^2 = 2x$. 5

b) Apply Charpit's method to find the complete integral of $px+qy=pq$. 5

Group - C

Answer either Q. **6** or Q. **7** and either Q. No. **8** or Q. No. **9**. 13 + 12 = 25

6. a) Prove that every extreme point of the convex set of all feasible solutions of the system $Ax=b$, $x \geq 0$ corresponds to a basic feasible solution of the system. 6

b) Show that $x_1 = 2$, $x_2 = 1$, $x_3 = 3$ is feasible solution of the system of equations
$$4x_1 + 2x_2 - 3x_3 = 1$$
$$6x_1 + 4x_2 - 5x_3 = 1$$
Reduce it to a basic feasible solution of the system. 7

7. a) Find the dual of the following primal problem :

Maximize $Z = 2x_1 + 3x_2$

subject to
$$-x_1 + 2x_2 \leq 4$$
$$x_1 + x_2 \leq 6$$
$$x_1 + 3x_2 \leq 9$$
and $x_1, x_2 \geq 0$

By solving the dual find the optimal solution of the primal problem. 7

b) Solve graphically the following rectangular game with pay-off matrix : 6

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 3 & 2 & -1 & 4 \\ 2 & 5 & 6 & -2 \end{bmatrix} \end{array}$$

8. a) Find an optimal solution of the following minimization problem : 6

	D_1	D_2	D_3	D_4	
O_1	19	20	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
	5	8	7	14	

b) Reduce the following pay-off matrix to a 2×2 matrix by dominance property and then solve the game problem, where A is the maximising player and B is the minimising player : 6

$$A \quad \overset{B}{\begin{bmatrix} 2 & 2 & 1 & -2 & -3 \\ 4 & 3 & 4 & -2 & 0 \\ 5 & 1 & 2 & 5 & 6 \\ 1 & 2 & 1 & -3 & 3 \end{bmatrix}}$$

9. a) Solve following travelling salesman problem : 6

	A	B	C	D
A	$\propto$	12	10	15
B	16	$\propto$	11	13
C	17	18	$\propto$	20
D	13	11	18	$\propto$

b) Solve the following assignment problem. 6

	I	II	III
A	11	23	16
B	22	25	19
C	29	13	27

Group - D

Answer any *three* questions : $3 \times 15 = 45$

10. a) A particle describes a path, which is nearly a circle under the action of a central force $\phi(u)$, $(u = \frac{1}{r})$ with the centre at the centre of the circle. Find the condition that the motion may be stable. Also find the apsidal angle in this case. 8

b) A smooth parabolic tube is placed, vertex downwards, in a vertical plane. A particle slides down the tube from rest under the influence of gravity. Prove that, in any position, the reaction of the tube is $\frac{2w\ (h+a)}{\rho}$, where w is the weight of the particle, ρ is the radius of curvature, $4a$ is the latus rectum and h is the original height of the particle above the vertex. 7

11. a) Find the radial and cross radial components of velocity and acceleration of a particle moving in a plane in polar coordinate. 7

b) A particle rests in equilibrium under the attraction of two centres of forces which attract directly as the distance, their attraction per unit of mass at unit distance being μ and μ', the particle slightly displaced towards one of them. Show that the time of small oscillation is $\frac{2\mu}{\sqrt{\mu+\mu'}}$. 8

12. a) Find the law of force to the pole when the path is the cardioide $r = a\ (1-\cos\theta)$ and prove that if F be the force at the apse and V be the velocity then $3V^2 = 4aF$. 7

b) A straight smooth tube turns about one extremity O in a horizontal plane with uniform angular velocity ω. Originally a particle is placed in the tube at a distance a from O and projected towards O with a velocity V. Show that if, $\omega < \frac{V}{a}$, the particle will reach O in time $\frac{1}{\omega}\tanh^{-1}\frac{a\omega}{V}$. 8

13. a) A particle is moving in a straight line with an acceleration $n^2 \times$ (distance) towards a fixed point in the line, in a medium which offers a resistance proportional to velocity and is simultaneously acted on by a periodic disturbing force $F \cos pt$ per unit mass. Discuss the motion. 7

b) A particle moves with a central acceleration $\mu\left(r + \frac{a^4}{r^3}\right)$ being projected from an apse at a distance a with a velocity $2\sqrt{\mu}\ a$. Prove that it describes the curve $r^2(2+\cos\sqrt{3}\ \theta) = 3a^2$. 8

14. a) A heavy particle slides down a rough cycloid whose base is horizontal and vertex downwards. Show that if it starts from rest at the cusp and comes to rest at the vertex, then $\mu^2 e^{\mu\pi} = 1$. 7

b) If the velocity of a body in an elliptic orbit, major axis $2a$, is the same at a certain point P, whether the orbit being described in a periodic time T about one focus S or in periodic time T' about other focus S', then prove that

$$SP = \frac{2aT'}{T + T'} \text{ and } S'P = \frac{2aT}{T + T'}.$$ 8
