West Bengal State University

.A./B.Sc./B.Com (Honours, Major, General) Examinations, 2014

PART – III

MATHEMATICS — HONOURS

Paper – VII

ration : 4 Hours] [Full Marks : 100

The figures in the margin indicate full marks.

Group – A

[Marks : 10]

Answer any *one* question. $1 \times 10 = 10$

a) If $\vec{F} = 2y\hat{i} - z\hat{j} + x^2\hat{k}$ and S is the surface of the parabolic cylinder $y^2 = 8x$ in the first octant bounded by the planes $y = 4$ and $z = 6$, evaluate $\iint_S \vec{F}.\hat{n}\,dS$, where $\hat{n}$ is the outward normal to the surface S. 5

b) Verify the divergence theorem for the vector function $\vec{F} = 2xz\hat{i} + y^2\hat{j} + yz\hat{k}$ taken over the surface of the cube bounded by $x = 0$, $x = 1$; $y = 0$, $y = 1$; $z = 0$, $z = 1$. 5

a) Evaluate $\oint_\Gamma [(\cos x \sin y - xy)\,dx + \sin x \cos y\,dy]$ by Green's theorem, where Γ is the circle $x^2 + y^2 = 1$ described in the positive sense. 5

b) Evaluate $\oint_\Gamma [xy\,dx + xy^2 dy]$ by Stokes theorem, where Γ is the square in the xy plane with vertices $(1, 0), (-1, 0), (0, 1), (0, -1)$. 5

Group – B

[Marks : 35]

Answer any *five* questions. $5 \times 7 = 35$

Three forces P, Q, R acting along the sides of a triangle formed by the lines $x + y = 3$, $2x + y = 1$ and $x - y + 1 = 0$. Find the equation of the line of action of the resultant.

4. A solid hemisphere rests in equilibrium with its curve surface in contact with a rough plane inclined to the horizon at an angle α. If $\alpha < \sin^{-1}\left(\frac{3}{8}\right)$ and also less than the angle of friction, then show that the inclination of the plane base of the hemisphere to the horizontal is $\sin^{-1}\left(\frac{8}{3}\sin\alpha\right)$.

5. What is the energy test of stability ? Establish the energy test of stability for a rigid body with one degree of freedom only, in equilibrium under conservative forces.

6. A square framework, formed by uniform heavy rods of equal weight w, jointed together, is hung up by one corner. A weight w is suspended from each of the three lower corners and the shape of the square is preserved by a light rod along the horizontal diagonal. Find the thrust of the light rod and show that in the position of equilibrium it is equal to the total weight of the rods.

7. Find the centre of gravity of the arc of the curve $x^{2/3} + y^{2/3} = a^{2/3}$, lying in the first quadrant.

8. A solid frustum of a paraboloid of revolution of height h and latus rectum $4a$, rests with its vertex on the vertex of a paraboloid of revolution whose latus rectum is $4b$. Show that the equilibrium is stable if $h < \frac{3ab}{a+b}$.

9. Two particles of masses m and m' are connected by a string of length l resting on a smooth cycloid with its vertex upwards, and base horizontal. Prove that in the equilibrium position the distance of the particle of mass m from the vertex measured along the arc is $\frac{m'l}{m+m'}$.

10. Define Poinsot's central axis of a system of forces acting on a body and find its equation.

11. Two equal forces act along the generators of the same system of the hyperboloid $\frac{x^2+y^2}{a^2} - \frac{z^2}{b^2} = 1$, and cut the plane $z = 0$ at the extremities of perpendicular diameters of the circle $x^2 + y^2 = a^2$. Show that the pitch of the equivalent wrench is $\frac{a^2 b}{a^2 + 2b^2}$.

Group – C

[Marks : 30]

(Rigid Dynamics)

Answer any *two* questions. $2 \times 15 = 30$

a) State D'Alembert's principle and deduce the equations of motion of the centre of inertia of a rigid body and the equations of motion relative to the centre of inertia. 8

b) The lengths AB and AD of the sides of a rectangle $ABCD$ are $2a$ and $2b$. Show that the inclination to AB of one of the principal axes at A is $\frac{1}{2}\tan^{-1}\frac{3ab}{2(a^2-b^2)}$. 7

a) A solid homogeneous cone of height h and semivertical angle α, oscillates about a diameter of its base. Show that the length of the simple equivalent pendulum is $\frac{h}{5}\left(2+3\tan^2\alpha\right)$. 7

b) A homogeneous sphere of radius a, rotating with angular velocity ω about a horizontal diameter, is gently placed on a table whose coefficient of friction is μ. Show that there will be slipping at the point of contact for a time $\frac{2\omega a}{7\mu g}$ and that then the sphere will roll with angular velocity $\frac{2\omega}{7}$. 8

a) If there exists a straight line such that the sum of the moments of external impulses acting on a system of particles about the line vanishes, show that the total angular momentum of the system about that line remains unaltered. 8

b) Two uniform rods AB and AC of masses m and m' respectively are freely jointed at A and laid on a smooth horizontal table in such a way that $\angle BAC$ is at right angle. The rod AB is struck by a blow P at B in a direction perpendicular to AB. Show that the initial velocity of A is $\frac{2P}{4m'+m}$. 7

Group – D

[Marks : 25]

Answer any *one* question from each Section.

Section – I

15. a) Find the condition for existence of metacentre of a body and prove that $HM = \frac{AK^2}{V}$, which is the formula for finding the metacentre of a body floating freely in a liquid at rest under gravity, where notations have their usual meaning. 8

b) A solid body consists of a right cone joined to hemisphere on the same base and floats with the spherical portion partly immersed. Prove that the greatest height of the cone consistent with stability is $\sqrt{3}$ times the radius of the base. 7

16. a) A quadrant of a circle is immersed in a liquid with a bounding radius in the surface. Find the position of its centre of pressure. 7

b) A conical vessel of height h and vertical angle 2α, contains water whose volume is one-half that of the cone. If the vessel and the contained water revolve with uniform angular velocity ω and no water overflows, show that ω must not be greater than $\sqrt{\frac{2g}{3h}} \cot\alpha$. 8

Section – II

17. a) A hollow gas-tight balloon containing helium, weighs W lbs when its lowest point touches the ground. It requires a force of w lbs to prevent it from rising. Show that it can float in equilibrium at a height $H \log\left(1 + \frac{w}{W}\right)$, where H is the height of homogeneous atmosphere. 5

b) Show that the pressure at a point in a fluid in equilibrium is the same in every direction. 5

18. a) A given volume V of liquid is acted upon by forces $-\frac{\mu x}{a^2}, -\frac{\mu y}{b^2}, -\frac{\mu z}{c^2}$.

Find the equation to the free surface. 5

b) An area is bounded by two concentric semicircles with common bounding diameter in the surface of a liquid. Show that the depth of its C.P. is $\frac{3\pi}{16} \frac{(a+b)(a^2+b^2)}{a^2+ab+b^2}$, where a and b are the radii of the semicircles. 5