West Bengal State University

B.A./B.Sc./B.Com. (Honours, Major, General) Examinations, 2014

PART-III

MATHEMATICS – Honours

Paper– VI

Duration : 4 Hours **Full Marks : 100**

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Use separate answer sheets for different Groups.

Answer any *two* questions from Q. Nos. 1 to 3 and any one from Q. Nos. 4 and 5.

Group-A

[Marks : 50]

1. Answer any *three* of the following : $3 \times 5 = 15$

a) Explain the term "Statistical Regularity". State analytically the frequency definition of probability on the basis of Statistical Regularity and hence deduce the following for an event A.

(i) $0 \leq P(A) \leq 1$

(ii) $P(\overline{A}) = 1 - P(A)$, $\overline{A}$ is complement of A.

b) If $\{A_n\}$ be a monotonic sequence of random events, then prove that $P\left(\lim\limits_{n \to \infty} A_n\right) = \lim\limits_{n \to \infty} P(A_n)$

c) The digits 1, 2, 3, 4, 5, 6, 7 are written in random order to form a seven digit number. What is the probability that it is divisible by 4.

d) A player tosses a coin and he scores one point for every head turned up and two for every tail. He will play until his score reaches or crosses n. If P_n is the probability of attaining exactly n points, then show that $p_n = \frac{1}{2}\left\{ p_{n-1} + p_{n-2} \right\}$. Hence find p_n and the limit of P_n as $n \longrightarrow \infty$.

e) If A and B be any two events, then show that

$$P\left(\overline{A} + B\right) = 1 - P(A) + P(AB).$$

Answer any *three* of the following : 3 × 5 = 15

a) In a Bernoullian sequence of n trials with probability of success p in each trial, show that the probability of at most k successes is given by

$$\frac{\int_0^{1-p} x^{n-k-1} \cdot (1-x)^k \, dx}{\int_0^1 x^{n-k-1} (1-x)^k \, dx}.$$

b) Use Tchebycheff's inequality to show that for $n \geq 36$, the probability that in n throws of a fair die the number of sixes lies between $\frac{n}{6} - \sqrt{n}$ and $\frac{n}{6} + \sqrt{n}$ is at least $\frac{31}{36}$.

c) If X_n is a Binomial (n, p) variate, then show that $\frac{X_n - np}{\sqrt{npq}}$, ($q = 1 - p$), is asymptotically normal (0, 1).

d) Find the median and mode of a distribution having probability density function $\lambda \cdot e^{-\lambda x}$, ($x > 0$).

e) From a point on a circle of radius r, chords are drawn at random. Find the expected value of the length of chords and the variance of the length of chords.

3. Answer any *three* of the following questions : $3 \times 5 = 15$

a) The joint density function of the random variables X, Y is given by $f(x, y) = 2, (0 < x < 1, 0 < y < x)$. Find the marginal and conditional definity functions and also compute $P(\frac{1}{4} < X < \frac{3}{4}) \mid Y = \frac{1}{2})$.

b) Define moment generating function of a random variable X. A continuous function has p.d.f. $f(x) = a\, exp(-ax)$ $(0 < x < \infty, a > 0)$. Obtain m.g.f. and hence find $E(X^n)$, n is a positive integer.

c) Write down the least square regression lines of Y on X and of X on Y. Show that the acute angle between these lines is $\tan^{-1}\left(\frac{1-\rho^2}{\rho^2} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\right)$ (symbols have their usual meanings). What happens when $\rho = \pm 1$?

d) Define concept of convergence in probability. Let $X_n \xrightarrow[in\,p]{} a$ as $n \to \infty$ and $Y_n \xrightarrow[in\,p]{} b$ as $n \to \infty$, then show that $X_n Y_n \xrightarrow[in\,p]{} ab$ as $n \to \infty$.

e) A random variable X has probability density function $12x^2(1-x)$, $(0 < x < 1)$. Compute $P(\mid X - m \mid \geq 2\sigma)$ and compare it with the limit given by Tchebycheff's inequality, where m is the mean and σ is the standard deviation of the distribution.

a) Obtain the maximum likelihood estimator σ^2 when μ is known where μ and σ are mean and standard deviation of a normal population. Show that this estimator is unbiased. 5

b) For a normal population show that the sampling distribution of the statistic $\chi^2 = \frac{nS^2}{\sigma^2}$ is χ^2-distribution with ($n - 1$) degrees of freedom, where n, S^2 and σ^2 are respectively the sample size, sample variance and population variance. 7

c) Define 'critical region' for testing a statistical hypothesis. The random variable X denoting the amount of consumption of a commodity follows the distribution

$$f(x, \theta) = \frac{1}{\theta} \cdot e^{-\frac{x}{\theta}}, \; 0 < x < \infty, \; \theta > 0.$$

The hypothesis $H_0 : \theta = 5$ is rejected in favour of $H_1 : \theta = 10$, if 15 units or more, chosen randomly be consumed. Obtain the size of the two types of errors and the power of the test. 8

a) Given $X = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$, where X_is ($i = 1, 2 \ldots, n$) are a sample of size n from a normal population (m, σ). Find the distribution of the statistic :

$$t = \sqrt{n}\left(\overline{X} - m\right)/s.$$ 8

b) The heights of 10 males of a normal population are found to be 70, 67, 62, 67, 61, 68, 70, 64, 65, 66 inches. Is it reasonable to believe that the average height is greater than 64 inches ? Test at 5% significance level assuming that for 9 degrees of freedom $P(t > 1{\cdot}83) = 0{\cdot}05$. 7

c) Explain the method of maximum likelihood. Estimate the parameter α of a continuous population having density function $(1+\alpha).x^{\alpha}$, $0 < x < 1$, by the above method. 5

Group-B

[Marks : 50]

Answer any three questions from Section-I and any two from Section-II.

Section-I

[Marks : 30]

6. a) What are the basic sources of errors in numerical computations of mathematical problems ? What do you mean by the term 'correct digit' of an approximate number ? Give suitable examples. 5

b) Define divided difference of two arguments x_0, x_1 and prove that

$$f(x_0, x_1, \ldots, x_n) = \sum_{i=0}^{n} \frac{f(x_i)}{(x_i - x_0)(x_i - x_1)\ldots(x_i - x_{i-1})(x_i - x_{i+1})\ldots(x_i - x_n)}. \quad 1+4$$

7. a) Obtain Lagrange's interpolation formula (without error term). 5

b) Explain the method of fixed point iteration for numerical solution of an equation of the form $x = \phi(x)$ and derive the condition of convergence. 5

8. a) Describe Gauss' elimination method for numerical solution of a system of linear equations and explain, in particular, the pivoting process involved in this method. 5

b) Explain the Regula-Falsi method for computing a simple real root of the equation $f(x) = 0$ and give a geometrical interpretation of the method. 5

a) State the basic concepts of Hermite Interpolation and prove the uniqueness of the Hermite Interpolation polynomial. 5

b) State the general principle of Newton-Cotes formula for evaluating an integral of the form $\int_a^b f(x)\,dx$, where a, b are finite. Hence or otherwise obtain the trapezoidal rule. Is it a closed type formula ? 1 + 3 + 1

0. a) Derive the modified Euler's formula to solve the differential equation $\frac{dy}{dx} = f(x, y)$, given that $y(x_0) = y_0$. 5

b) Use fourth order Runge-Kutta method to solve $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0) = 1$ for $x = 0{\cdot}0\ (0{\cdot}1)\ 0{\cdot}2$ correct up to four decimal places. 5

Section-II

[Marks : 20]

1. Answer any *five* of the following : 5 × 2 = 10

(i) Convert $(A35)_{16}$ into binary.

(ii) Define source program and object program.

(iii) Name the five functional units of a computer.

(iv) What is compiler ?

(v) If $f(x, y, z) = (x' + y' + z)(x + y' + z')(x' + y + z')$, express $f'(x, y, z)$ in DNF.

(vi) Find the CNF of $ab + a'b$.

(vii) Use 2's complement to compute $10100.01_2 - 11011.10_2$

(viii) Draw the flowchart to find $\underline{\lfloor n}$.

12. a) Write a Fortran 77 or C program for bisection method to find a real root of $e^{x^2} + \log_3(x+2) - 5(x+1) = 0$. 5

b) Write a Fortran 77 or C program to read n numbers and print the smallest of them. 5

13. a) Write a Fortran 77 or C program to find the sum of k terms of the divergent series $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots$ when the sum exceeds x. 5

b) Write a Fortran 77 or C program which will read a four-digit positive integer m and give the sum of the digits of m as the output. 5
