West Bengal State University

B.A./B.Sc./B.Com. (Honours, Major, General) Examinations, 2012

PART-III

MATHEMATICS - (HONOURS)

Paper– VII

Duration : 4 Hours Full Marks

Candidates are required to give their answers in their own words as far as practicabl

The figures in the margin indicate full marks.

GROUP – A

(Marks – 10)

Answer any *one* question. 1 × 1

1. a) Show that the area bounded by a simple closed curve C is give $\frac{1}{2}\oint (x\,dy - y\,dx)$.

   b) Prove that $\iint_S \vec{r} \times d\vec{S} = 0$ for any closed surface S.

   c) Prove that $\iiint_V \frac{dV}{r^2} = \iint_S \frac{\vec{r}\,.\,\vec{n}}{r^2}\,dS$ where S is any closed surface enclo volume V. 3

2. a) Verify Stokes' Theorem for

   $\vec{a} = (y - z + 2)\,\vec{i} + (yz + 4)\,\vec{j} - xz\vec{k}$, where S is the surface of the cube :

   $x = 0,\ y = 0,\ z = 0;$

   $x = 2,\ y = 2,\ z = 2$ above the xy-plane.

   b) Evaluate $\oint \{(x^2 - 2xy)\,dx + (x^2y + 3)\,dy\}$ around the boundary region defined by $y^2 = 8x$ and $x = 2$ by using Green's Theorem.

GROUP – B

(Marks : 35)

Answer any five questions : $5 \times 7 = 35$

3. Investigate the condition of equilibrium of a particle constrained to rest on a rough plane curve and on a rough surface under the action of any given forces. 7

4. The density at any point of a circular lamina varies as the n^{th}-power of the distance from a point O on the circumference. Show that the centre of gravity of the lamina divides the diameter through O in the ratio $(n + 2) : 2$. 7

5. Two equal uniform rods AB and AC, each of length $2b$, are freely jointed at A and rest on a smooth vertical circle of radius a. Show that, if 2θ be the angle between them, then $b \sin^3\theta = a\cos\theta$. 7

6. A string of length a, forms the shorter diagonal of a rhombus formed of four uniform rods, each of length b and weight w, which are hinged together. If one of the rods be supported in a horizontal position, prove that the tension of the string is $\dfrac{2w\,(2b^2 - a^2)}{b\sqrt{4b^2 - a^2}}$. 7

7. What is the energy test of stability ? Establish the energy test of stability for a rigid body with one degree of freedom only, in equilibrium under conservative forces. 7

8. A uniform smooth rod passes through a ring at the focus of a fixed parabola whose axis is vertical and vertex below the focus, and rests with one end on the parabola. Prove that the rod will be in equilibrium if it makes with the vertical an angle θ given by the equation $\cos^4\dfrac{\theta}{2} = \dfrac{a}{2c}$, where $4a$ is the latus rectum of the parabola and $2c$ the length of the rod. Investigate the stability of equilibrium in this position. 7

9. Show that the necessary and sufficient condition for a system of coplanar forces in equilibrium, to be in a static equilibrium is that the virial of the system vanishes. 7

10. Prove that any system of forces acting on a rigid body can be reduced to a single force and a single couple whose axis lies along the line of action of the force. Find the equation of this line of action of the force. 7

11. Two forces $2P$ and P act along the lines whose equations are $y = x\tan\alpha$, $z = c$ and $y = -x\tan\alpha$, $z = -c$ respectively. Find the equation of the central axis. 7

GROUP- C

(Marks : 30)

Answer any two questions. $2 \times 15 =$

12. a) Find whether a given straight line is at any point of its length, a principal axis o given material system. If so, find the directions of the other two principal ax Hence show that if an axis passes through the centre of inertia of a body and is principal axis at some point of its length, then it is a principal axis at all points its length.

b) A weightless straight rod ABC of length $2a$ is movable about the end A which fixed and carries two particles of the same mass, one fastened to the middle poi B and the other to the end C of the rod. If the rod be held in a horizontal positi and then let go, show that its angular velocity when vertical is $\sqrt{\frac{6g}{5a}}$, and that $\frac{5}{}$ is the length of the simple equivalent pendulum. 8 +

13. a) A uniform sphere of radius a is rotating about a horizontal diameter with angul velocity Ω and is gently placed on a rough plane which is inclined at an angle α the horizontal, the sense of the rotation being such as to tend to cause the sphe to move up the plane along the line of greatest slope. Show that if the coefficient friction be $\tan\alpha$, the centre of the sphere will remain at rest for a time $\frac{2a\Omega}{5g\sin}$ and will then move downwards with acceleration $\frac{5}{7} g \sin\alpha$.

b) Find the moment of momentum of a rigid body moving in two-dimensions, abo the origin. 8 +

14. a) Two like rods AB and BC each of length $2a$ are freely jointed at B; AB can tu round the end A and C can move freely on a vertical straight line through Initially the rods are held in a horizontal line, C being in coincidence with A, an they are then released. Show that when the rods are inclined at an angle θ to th horizontal, the angular velocity of either is $\sqrt{\frac{3g}{a} \frac{Sin\theta}{1+3\cos^2\theta}}$.

b) An elliptic area of eccentricity e is rotating in its own plane about one of the fo with angular velocity ω. This focus is set free and the other focus is fixed at th same instant. Show that the ellipse now rotates about it with angular velocit $\omega \cdot \frac{2-5e^2}{2+3e^2}$. 8 +

GROUP-D

(Marks : 25)

Answer *one* question from each Section :

SECTION – I

15. a) A mass of liquid is in equilibrium under the action of conservative system of forces. Show that the surface of equi-pressure, equi-density, and equi-potential energy coincide. If the system of forces is the force of gravity only, show that these surfaces are horizontal.

b) A square lamina is wholly immersed in a heavy homogeneous fluid with its plane vertical and one corner in the surface. If it be turned in its own plane about this corner, and is always immersed, show that the locus of the centre of pressure in the lamina is a straight line. 8 + 7

16. a) Find the pressure in an isothermal atmosphere at a height Z when (i) the gravity g is constant and (ii) when the gravity g is variable.

b) If a floating solid be a cylinder, with its axis vertical, the ratio of whose specific gravity to that of the fluid is σ , prove that the equilibrium will be stable, if the radius of the base to the height is greater than $[\,2\sigma\,(\,1-\sigma\,)\,]^{\frac{1}{2}}$. 8 + 7

SECTION – II

17. a) Show that the pressure at a point in a fluid in equilibrium is the same in every direction.

b) A hollow right circular cone has its vertex uppermost and base horizontal and is just full of a homogeneous liquid. Show that the resultant thrust of the liquid on the curved surface of the cone is $\frac{2}{3}$ times the total thrust on the base of the cone.

5 + 5

18. a) A body floats partially immersed in a liquid and is free to turn about a fixed point O of the body. Find the necessary and sufficient conditions of equilibrium of the body.

b) Taking into account the variation of gravity with height and assuming that the temperature of the air is constant at all heights, prove that at height x the pressure p of the air is given by $\log\frac{p}{p_0} = -\frac{g_0 ax}{k(a+x)}$, where a is the radius of he earth, $k = \frac{p_0}{\rho_0}$, and p_0, ρ_0, g_0 are respectively the air pressure, air density and gravity on earth's surface. 5 + 5