West Bengal State University

B.A./B.Sc./B.Com. (Honours, Major, General) Examinations, 2012

PART – II

MATHEMATICS — HONOURS

Paper – IV

Duration : 4 Hours] [Full Marks : 100

The figures in the margin indicate full marks.

GROUP – A

Answer any *two* questions. $2 \times 10 = 20$

1. a) From points on the circle $x^2 + y^2 = a^2$ tangents are drawn to the hyperbola $x^2 - y^2 = a^2$. Prove that the locus of the middle points of the chords of contact is the curve $(x^2 - y^2)^2 = a^2 (x^2 + y^2)$. 5

   b) If the points of the intersection of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and $\dfrac{x^2}{\alpha^2} + \dfrac{y^2}{\beta^2} = 1$ be the ends of the conjugate diameters of the former, prove that $\dfrac{a^2}{\alpha^2} + \dfrac{b^2}{\beta^2} = 2$. 5

2. a) Show that the conditions that the lines of section of the plane $lx + my + nz = 0$ and the cones $fyz + gzx + hxy = 0$, $ax^2 + by^2 + cz^2 = 0$ should be coincident are $\dfrac{bn^2 + cm^2}{fmn} = \dfrac{cl^2 + an^2}{gnl} = \dfrac{am^2 + bl^2}{hlm}$. 5

   b) Find the locus of a luminous point if the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$ casts a circular shadow on the plane $z = 0$. 5

3. a) If the generator through a point P on the hyperboloid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = 1$ meets the principal elliptic section in two points such that the eccentric angle of one is three times that of the other, prove that P lies on the curve of intersection of the hyperboloid with the cylinder $y^2 (z^2 + c^2) = 4b^2 z^2$. 5

7. a) Prove that if either the primal or the dual problem has a finite optimal solution, then the other problem will also have a finite optimal solution and the optimal values of the objective functions in both the problems will be same. 6

b) Solve the following L.P.P. using duality theory,

Maximize $Z_x = 4x_1 + 3x_2$

subject to $x_1 \quad \leq 6$

$x_2 \quad \leq 8$

$x_1 + x_2 \quad \leq 7$

$3x_1 + x_2 \quad \leq 15$

$-x_2 \quad \leq 1,\ x_1, x_2 \geq 0$ 7

8. a) Prove that in each transportation problem, there exists at least one B.F.S. which makes the objective function a minimum. 3

b) Find the initial B.F.S. of the following transportation problem by matrix minima method and then solve the problem : 5

	D_1	D_2	D_3	D_4	ai
O_1	5	3	6	2	19
O_2	4	7	9	1	37
O_3	3	4	7	5	34
bj	16	18	31	25	

b) Reduce the equation $3x^2+7y^2+z^2+10yz-2zx+10xy+4x-12y-4z+1=$ the standard form and state the nature of the conicoid.

GROUP – B

Answer any *one* question. 1 × 1

4. a) Find the eigenvalues and eigenfunctions of the differential equ

$$\frac{d^2y}{dx^2}+\lambda y=0 \quad (\lambda>0)$$ with the following boundary cond

$$y(0)+y'(0)=0;\ y(1)+y'(1)=0.$$

b) Solve $\frac{dx}{dt}-3x-6y=t^2$

$$\frac{dy}{dt}+\frac{dx}{dt}-3y=e^t$$

5. a) Find a complete integral of $xpq+yq^2=1$ by Charpit's method.

b) Solve $y^2zp+x^2zq=xy^2$ by Lagrange's method.

GROUP – C

Answer either Question No. **6** *or* Question No. **7**

and either Question No. **8** *or* Question No. **9**. 13 + 1

6. a) Prove that if a linear programming problem admits of an optimal solution, the objective function assumes its optimum value at an extreme point c convex set generated by the feasible solutions and also prove that if the has at least two optimal feasible solutions, then there are infinite numb optimal solutions.

b) Solve the L.P.P.

Maximize $Z=x_1+2x_2$

subject to $x_1-5x_2\le 10$

$$2x_1-x_2\ge 2$$
$$x_1+x_2=10\ ,\ x_1,x_2\ge 0$$

c) Find the optimal assignment and the corresponding assignment cost from following cost matrix :

$$\begin{array}{c|ccccc|}
 & A & B & C & D & E \\
\hline
1 & 9 & 8 & 7 & 6 & 4 \\
2 & 5 & 7 & 5 & 6 & 8 \\
3 & 8 & 7 & 6 & 3 & 5 \\
4 & 8 & 5 & 4 & 9 & 3 \\
5 & 6 & 7 & 6 & 8 & 5 \\
\hline
\end{array}$$

9. a) Solve the following game problem by converting it into L.P.P. :

$$\begin{array}{cc}
 & \text{Player } B \\
 & \begin{array}{cc} B_1 & B_2 \end{array} \\
\text{Player } A \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}
\end{array}$$

b) Let $[a_{ij}]_{m \times n}$ be the pay-off matrix for a two person zero-sum game. Then, that $\min\limits_j \max\limits_i a_{ij} \geq \max\limits_i \min\limits_j a_{ij}$.

c) Solve the game problem by reducing it into 2×2 problem with the h dominance property :

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
4 & 2 & 0 & 2 & 1 \\
4 & 3 & 1 & 3 & 2 \\
4 & 3 & 4 & -1 & 2
\end{pmatrix}$$

GROUP – D

Answer any *three* questions. $3 \times 15 = 45$

10. a) A heavy particle is attached to the lower end of an elastic string, the upper end of which is fixed. The modulus of elasticity of the string is equal to the weight of the particle. The string is drawn vertically down till it is four times its natural length and is then let go. Show that the particle will return to this point in time $\sqrt{\dfrac{a}{g}}\left(2\sqrt{3}+\dfrac{4\pi}{3}\right)$, where a is the unstretched length of the string. 7

b) Find the radial and cross-radial components of velocity and acceleration of a particle moving in a plane in polar co-ordinates (r , θ). 8

11. a) A particle is projected under gravity at an angle α with the horizontal in a medium which produces a retardation equal to k times the velocity. It strikes the horizontal plane through the point of projection at an angle ω and the time of flight is T. Prove that $\dfrac{\tan\omega}{\tan\alpha} = \dfrac{e^{kT} - 1 - kT}{e^{-kT} - 1 + kT}$. 7

b) A particle describes a path which is nearly a circle under the action of a central force $\phi(u)$, $\left(u = \dfrac{1}{r}\right)$ with the centre at the centre of the circle. Find the condition that the motion may be stable. Also find the apsidal angle in this case. 8

12. a) A particle moves under a force which is always directed towards a fixed [

and equal to $\mu \div (\text{distance})^2$ per unit mass. Show that its path is a conic se

and distinguish between the three cases that arise.

b) A planet is describing an ellipse about the sun as focus; show that its ve

away from the sun is greatest when the radius vector to the planet is at

angle to the major axis of the path and it is $\dfrac{2\pi ae}{T\sqrt{1-e^2}}$ where $2a$ is the major

e the eccentricity and T the periodic time.

13. a) Two smooth spheres of masses m_1 and m_2 moving with respective vel

u_1 and u_2 at angles α_1 and α_2 with their line of centres, impinge oblique

be the coefficient of restitution, then show that the loss of K.E. is

$$\frac{1}{2}\frac{m_1 m_2}{m_1+m_2}\left(1-e^2\right)\left(u_1\cos\alpha_1 - u_2\cos\alpha_2\right)^2$$

b) A point moves along the arc of a cycloid in such a manner that the direc

motion rotates with constant angular velocity; show that the acceleration

moving point is constant in magnitude.

14. a) A particle is projected with velocity $\sqrt{\dfrac{2\mu}{3a^2}}$ at right angle to the radius vector at a distance a from a centre of attracting force $\dfrac{\mu}{r^4}$ per unit mass, r being the distance of the particle from the centre of force. Find the path of the particle and show that the time it takes to reach the centre of force is $\dfrac{3\pi}{8}\sqrt{\dfrac{3a^5}{2\mu}}$. 8

b) A planet is describing an elliptical orbit round the sun. When the planet is at perihelion, the mass of the sun is suddenly doubled. Show that the planet will continue to describe an elliptical orbit, but that its velocity at perihelion is to its former velocity at aphelion is twice the major axis of the former orbit is to the major axis of the new one. 7