West Bengal State University

B.A./B.Sc./B.Com. (Honours, Major, General) Examinations, 201

PART – I

MATHEMATICS (Honours)

Paper – I

Duration : 4 Hours] [Full Marks

The figures in the margin indicate full marks.

GROUP - A

Answer any *five* questions. 5 × 5

1. i) If P_n is the n th prime, then show that $\frac{1}{P_1}+\frac{1}{P_2}+\ldots+\frac{1}{P_n}$ is not an integer.

   ii) If $gcd\,(\,a,\,b\,)=1$, show that $gcd\,(\,a+b,\,a-b\,)=1$ or 2, where a and b ar non-zero integers.

2 i) Use mathematical induction to show that $(5+\sqrt{11})^n+(5-\sqrt{11})^n$ is an integer, for all natural numbers n.

   ii) Prove that $19^{2002}\equiv -1\,(\bmod 181)$

3. i) If $n=p_1^{a1}p_2^{a2}\ldots p_r^{ar}$, where $p_1,p_2,\ldots p_r$ are prime to each other, then prove

$$\varphi(n)=n\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\ldots\left(1-\frac{1}{p_r}\right).$$

   ii) Show that the product of any 3 consecutive integers is divisible by 6.

4. Prove that $\sin\left[i\log\frac{a-ib}{a+ib}\right]=\frac{2ab}{a^2+b^2}$.

5. i) Use De Moivre's theorem to show that for integral values of n,

$\alpha^n + \beta^n = 2^{\frac{n}{2}+1} \cos\frac{n\pi}{4}$, where α, β are the roots of $x^2 - 2x + 2 = 0$. 3

ii) Find all complex numbers z, such that $\exp(2z+1) = i$. 2

6. i) If z_1 and z_2 are complex numbers, then prove that

$\sin h(z_1 + z_2) = \sin h\, z_1 \cos h\, z_2 + \cos h\, z_1 \sin h\, z_2$. 3

ii) Show that different values of i^i can be so arranged that they form a G.P. 2

7. i) Use Sturm's theorem to show that $x^3 - 7x + 7 = 0$ has two roots between 1 and 2 and other root between – 3 and – 4. 3

ii) Find the multiple roots, if any, of the equation $x^4 + 3x^3 + 4x^2 + 3x + 1 = 0$. 2

8. i) Prove that the roots of the equation $\frac{1}{x+a_1} + \frac{1}{x+a_2} + \ldots + \frac{1}{x+a_n} = \frac{1}{2}$ are all real, where $a_1, a_2, \ldots a_n$ are all negative real numbers. 3

ii) If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, then find the value of

$\sum \frac{1}{\alpha^2 - \beta\gamma}$. 2

9. i) If the equation $x^4 + px^2 + qx + r = 0$ has three equal roots, then show that

$8p^3 + 27q^2 = 0$ and $p^2 + 12r = 0$. 3

ii) Find the equation whose roots are squares of the roots of the equation

$x^4 - x^3 + 2x^2 - x + 1 = 0$. 2

GROUP - B

Answer any *two* questions. 2 × 10

10. i) Let A, B and C denote the subsets of a set S and C' denote the complement in S. If $A \cap C = B \cap C$ and $A \cap C' = B \cap C'$, then prove that $A = B$.

ii) Examine whether the relation ρ is an equivalence relation on the set S integers $\rho = \{(a, b) \in S \times S : |a - b| \leq 3\}$.

iii) Define injective and surjective mappings. Give an example of an inj mapping which is not surjective. Let $f : A \to B$, $g : B \to C$, $h : B \to C$ be mappings such that f is surjective and $gof = hof$. Prove that $g = h$. 1 +

11. i) Show that an equivalence relation on a set S determines a partition of S.

ii) If $f : S \to T$ is one-one onto, then prove that $f^{-1} : T \to S$ is one-one onto.

iii) Prove that any finite semigroup in which both cancellation laws hold is a

12. i) In a group G, prove that $(a^{-1})^{-1} = a$ where $a \in G$ and hence show tha group of even order contains an element of order 2.

ii) In an Abelian group G, prove that $(ab)^n = a^n b^n$ for all $a, b \in G$ where integer.

iii) Prove that a non-empty subset H of a group G is a subgroup of G if and all $a, b \in H$, $a^{-1}b \in H$.

13. i) If $x^3 = x$ for all $x \in R$, then prove that R is a commutative ring.

ii) Prove that the characteristic of an integral domain is either zero or integer.

iii) Prove that a field does not contain any divisor of zero. Prove that the set of all 2×2 matrices $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ where a, b are real numbers, is not a field with respect to matrix addition and multiplication. 2 + 2

GROUP - C

Answer any *three* questions. $3 \times 5 = 15$

14. i) A and B are two real orthogonal matrices of same order and $\det A + \det B = 0$. Prove that $A + B$ is a singular matrix. 3

ii) If A be a Hermitian matrix, then show that iA is a skew- Hermitian matrix. 2

15. For a square matrix A of order n, prove that $A(adj\,A) = (adj\,A)A = (del.A)I_n$, where I_n denotes the identity matrix of order n. 5

16. Apply Laplace's method along second and third rows to prove that

$$\begin{vmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{vmatrix} = (a^2+b^2+c^2+d^2)^2$$ 5

17. If the adjugate of $\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$ is $\Delta' = \begin{vmatrix} A & H & G \\ H & B & F \\ G & F & C \end{vmatrix}$, then prove that $\dfrac{BC-F^2}{a} = \dfrac{GH-AF}{f} = \Delta$. 5

18. Find two non-singular matrices P and Q such that the matrix PAQ is the fully reduced normal form of the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{pmatrix}$. 5

19. Reduce the quadratic from $2x^2 - 4yz + 6zx + y^2 + z^2$ into its normal form and obtain the rank, signature and nature of the form. 5

GROUP - D

Answer any *one* question. 1 × 10 =

20. i) Solve graphically the following L.P.P. :

Minimize $Z = 3x + 4y$

subject to $5x + 4y \geq 20$

$-x + y \leq 3$

$x \leq 4$,

$y \geq 3$,

$x \geq 0$, $y \geq 0$

ii) Obtain the basic feasible solutions of the system of equations :

$x_1 + 4x_2 - x_3 = 5$

$2x_1 + 3x_2 + x_3 = 8$

21. i) Find all the basic solutions to the system of linear equations :

$2x_1 + x_2 - x_3 = 2$

$3x_1 + 2x_2 + x_3 = 3$

Are the solutions degenerate ? 4 +

ii) A furniture manufacturer wishes to determine the number of tables and chai to be made by him in order to optimize the use of his available resources. Thes products utilize two different types of timber and he has on hand 1,500 boar feet of the first type of 1,000 board feet of the second type. He has 800 ma hours available for the total job. Each table and chair requires 5 and 1 boar feet respectively of the first type of timber and 2 and 3 board feet of the secon type. 3 man hours are required to make a table and 2 man hours are needed t make a chair. The manufacturer makes a profit of Rs. 12 on a table and Rs. 5 on a chair. Write out the complete linear programming formulation of the problem in terms of maximizing the profit. 5

GROUP - E

Section - I

Answer any *three* questions. $3 \times 5 = 15$

22. Reduce the following conic to its canonical form and determine its nature :

$$11x^2 - 4xy + 14y^2 - 58x - 44y + 71 = 0$$

Also find its eccentricity and the length of the latus rectum. 5

23. i) Find the equation to the pair of straight lines passing through the origin, perpendicular to the pair of straight lines given by $2x^2 + 5xy + 2y^2 + 10x + 5y = 0$. 2

ii) Prove that the equation $x^2 + 6xy + 9y^2 + 4x + 12y = 5$ represents a pair of parallel straight lines and find the distance between them. 3

24. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines, then prove that the area of the triangle formed by the bisectors of the angles between them and the axis of x is $\frac{\sqrt{(a-b)^2 + 4h^2}}{2h} \cdot \frac{ca - g^2}{ab - h^2}$. 5

25. If PQ is a variable chord of the conic $\frac{l}{r} = 1 - e\cos\theta$ subtending constant angle 2β at the focus S where S is the pole, then show that the locus of the foot of the perpendicular from S on PQ is $r^2(e^2 \sec^2\beta) + 2elr\cos\theta + l^2 = 0$. 5

26. Prove that the locus of the poles of the normal chords of the hyperbola $xy = c^2$ is the curve $(x^2 - y^2)^2 + 4c^2xy = 0$. 5

Section - II

Answer any *three* questions. 3 × 5

27. Prove that, if the angle between two straight lines whose direction cosines are give $l+m+n=0$ and $fmn+gnl+hlm=0$ is $\frac{\pi}{3}$, then $\frac{1}{f}+\frac{1}{g}+\frac{1}{h}=0$.

28. Find the equation of the plane which passes through the point (2, 1, –1) ar perpendicular to each of the planes $x-y+z=1$ and $3x+4y-2z=0$.

29. i) Find the shortest distance between the lines $\frac{x+1}{2}=\frac{y-3}{4}=z+5$ $y-mx=z=0$. For what value of m will the two lines intersect ?

ii) Determine the value of k for which the straight lines $\frac{x-1}{2}=\frac{y-4}{1}=\frac{z-5}{2}$ $\frac{x-2}{-1}=\frac{y-8}{k}=\frac{z-11}{4}$ intersect. 3

30. i) Find the distance of the point (10, 1, 4) from the $2x+y-z+3=0=x-y+2z+3$.

ii) Show that the locus of a variable straight line which intersects the three line

$$y=4x, z=6;$$

$$y=-4x, z=-6; \text{ and}$$

$$y=z,\ 2x=-3$$

is the surface $y^2-16x^2=z^2-36$. 3

31. Show that the straight lines $\frac{x-5}{-1}=\frac{y-2}{1}=\frac{z+3}{2}$ and $x+y+5z-2=0=2x-y+4z$ are coplanar. Find also the equation of the plane.