WEST BENGAL STATE UNIVERSITY

Syllabus
For
THE THREE- YEAR B.Sc. HONOURS
COURSES OF STUDIES

MATHEMATICS

लक्ष्यं विश्वमानम्

Berunanpukuria, P. O. Malikapur,
Barasat , North 24 Parganas,
Kolkata- 700126, W.B.,
INDIA

2008 ONWARDS

COURSE STRUCTURE & DISTRIBUTION OF MARKS
for three-year B.Sc. Honours Course in Mathematics

PART-I :

AT THE END OF FIRST YEAR
(TWO PAPERS OF 100 MARKS EACH)

PAPER-I

Group A	Classical Algebra I	25 Marks
Group B	Modern Algebra I	20 Marks
Group C	Linear Algebra I	15 Marks
Group D	Introduction to Linear Programming	10 Marks
Group E	Analytical Geometry of Two & Three dimensions I	30 Marks

PAPER-II

Group A	Real Analysis I	25 Marks
Group B	Application of Differential Calculus and Evaluation of Integral I	20 Marks
Group C	Differential Equation I	30 Marks
Group D	Vector Algebra & Analysis I	25 Marks

PART-II :

AT THE END OF SECOND YEAR
(TWO PAPERS OF 100 MARKS EACH)

PAPER-III

Group A	Classical Algebra II	15 Marks
Group B	Modern Algebra II	10 Marks
Group C	Linear Algebra II	20 Marks
Group D	Real Analysis II	20 Marks
Group E	Function of Several Variables	25 Marks
Group F	Application of Integral Calculus II	10 Marks

PAPER-IV

Group A	Analytical Geometry of Two & Three dimensions II	20 Marks
Group B	Differential Equation II	10 Marks
Group C	Linear Programming and Game Theory	25 Marks
Group D	Analytical Dynamics of a Particle	45 Marks

2

PART-III :

AT THE END OF THIRD YEAR
(FOUR PAPERS OF 100 MARKS EACH)

PAPER-V

Group A	Real Analysis II	70 Marks
Group B	Metric Space	15 Marks
Group C	Complex Analysis	15 Marks

PAPER-VI

Group A	Probability & Statistics	50 Marks
Group B	Numerical Analysis & Computer Programming	50 Marks

PAPER-VII

Group A	Vector Analysis II	10 Marks
Group B	Analytical Statics	35 Marks
Group C	Rigid Dynamics	30 Marks
Group D	Hydrostatics	25 Marks

PAPER-VIIIA (50 MARKS)

Group A	Algebra III (Linear, Modern and Boolean)	25 Marks
Group B	Differential Equation III	15 Marks
Group C	Tensor Algebra and Calculus	10 Marks

PAPER-VIIIB

Practical	50 Marks

N.B. : Minimum number of classes required : 20 classes per week

DETAILED SYLLABUS : HONOURS
Syllabus for three-year B.Sc. Honours Course
(Figures in the margin indicate the minimum number of lectures required to cover the topic)

PART-I
Paper-I
Group-A (25 Marks)
(Classical Algebra I)

1. Integers (It is not the aim to give an axiomatic development of the topic, rather assume that the students are familiar with the set Z of integers, the elementary properties of addition, multiplication and order.)

    Statements of well ordering principle, first principle of mathematical induction, second principle of mathematical induction. Proofs of some simple mathematical results by induction. Divisibility of integers. The division algorithm ($a = gb + r$, $b \neq 0$, $0 \leq r < b$). The greatest common divisor (g.c.d.) of two integers a and b. (This number is denoted by the symbol (a, b)]. Existence and uniqueness of (a, b). Relatively prime integers. The equation $ax + by = c$ has integral solution iff (a, b) divides c. (a, b, c are integers).

    Prime integers. Euclid's first theorem: If some prime p divides ab, then p divides either a or b.

    Euclid's second theorem: There are infinitely many prime integers. Unique factorisation theorem. Congruences, Linear Congruences.

    Statement of Chinese Remainder Theorem and simple problems, Theorem of Fermat. Multiplicative function ϕ (n).   [15]

2. Complex Numbers: De-Moivre's Theorem and its applications, Exponential, Sine, Cosine and Logarithm of a complex number. Definition of a^z ($a \neq O$). Inverse circular and Hyperbolic functions.   [8]

3. Polynomials with real co-efficients : Fundamental theorem of classical Algebra (statement only). The n th degree polynomial equation has exactly n roots. Nature of roots of an equation (surd or complex roots occur in pairs). Statements of Descartes' rule of signs and of Sturm's Theorem and their applications. Multiple roots. Relation between roots and coefficients. Symmetric functions of roots. Transformation of equations.   [8]

Group-B (20 Marks)
(Modern Algebra I)

1. Basic concepts: Sets, Sub-sets, Equality of sets, and Operations on sets - Union, Intersection and Complement, Symmetric difference. Properties including De Morgan's laws. Cartesian products: Binary relations from a set to a set (domain, range, examples from R X R). Equivalence relation : Fundamental Theorem on Equivalence relation (Partition). Relation of Partial Order. Congruence relation module n is an Equivalence Relation. Congruence Classes. Mapping: Injection, Surjection, Bijection, Inverse and Identity mapping. Composition of Mappings and its Associativity. Binary operation : Intuitive definition. Definition on the basis of mapping. Binary operation in a finite set by Cayley Tables.   [10]

2. Introduction to Group Theory : Groupoid, Semi-group, Quasigroup, Monoid, Group. Definition with both-sided identity and Inverse. (Examples of finite and infinite groups taken from various branches, e.g. from number system, roots of unity, non-singular real matrices of a fixed order, symmetries of a square, triangle, etc.) Additive group of integers modulo an integer m, Multiplicative group of integers modulo a prime p. Klein's 4 Group. Properties deducible from the definition of group including solvability of $ax = b$ and $ya = c$. Any finite semi-group, in which both cancellation laws hold, is a group. Integral powers of an element and laws of indices in a group. Order of a group and order of an element of a group.

   Subgroups : Necessary and sufficient condition for a sub-set of a group to be a sub-group. Intersection and Union of two sub-groups. Necessary and sufficient condition for the union of two sub-groups to be a sub group.   [10]

3. Introduction to Rings and Fields :

   Ring : Definition and examples. Ring of integers modulo n. Properties directly following from the definition. Multiplicative identity in a ring. Commutative ring. Divisors of zero. Commutative ring with identity and without zero divisor - Integral Domain.

   Field : Definition and examples. Every field is an integral domain. Every finite integral domain is a field. Sub-ring and Sub-field. Necessary and sufficient condition of a sub-set of a ring (a field) to be a sub-ring (sub-field). Characteristic of a ring and of an integral domain.   [10]

Group-C (15 Marks)
(Linear Algebra I)

1. Matrices of real and complex numbers : Definition of a matrix. Equality of matrices. Addition, multiplication, Scalar multiplication. Transpose of a matrix. Symmetric, Skew-symmetric and Hermitian matrix. Orthogonal matrix. [5]

2. Determinants : Definition of a determinant of a square matrix. Basic Properties. Minors and Cofactors. Expansion of determinant. Laplace's method. Product of determinants. Symmetric and Skew-symmetric determinant. Vandermonde's determinant. Solution of the system of equations by Cramer's Rule (Problems of determinants of order greater than 4 will not be asked). (No proof of theorems). [8]

3. (a) Adjoint of a square matrix. For a square matrix A of order n,

   A. Adj A = Adj A. A = det $A.I_n$

   (b) Non-singular matrix iff corresponding determinant is non-zero. Non-singular matrix and Invertible Matrix.

   (c) Elementary operations. Echelon matrix. Rank of a matrix - Determination of rank of a matrix - statement and applications of all relevant results and theorems (No proof required). [5]

4. Normal forms. Elementary matrix : Statement and application of the results on Elementary matrix. The normal form and equivalence of matrices. Congruence of matrices - Statement and application of relevant theorems.

   Real Quadratic form involving three variables. Reduction to Normal Form (Statement and application of relevant theorems). [10]

Group-D (10 Marks)
(Introduction to Elements of Linear Programming)

1. Definition of L. P. P. Formation of L. P. P. from daily life involving inequations. Graphical solution of L.P.P.

   Basic solutions and Basic Feasible Solutions (BFS) with reference to L.P.P. Matrix formulation of L.P.P. Degenerate and Non-degenerate B.F.S. [8]

Group-E (30 Marks)
(Analytical Geometry of Two & Three Dimensions-I)

Two Dimensions (15 Marks)

1. a) Transformation of Rectangular axes : Translation, Rotation and their combinations. Theory of Invariants. [2]

    b) General Equation of second degree in two variables: Reduction into canonical form. Classification of conics, Lengths and position of the axes. [2]

2. Pair of straight lines : Condition that the general equation of second degree in two variables may represent two straight lines. Point of intersection of two intersecting straight lines. Angle between two lines given by $ax^2 + 2hxy + by^2 = 0$. Angle bisector. Equation of two lines joining the origin to the points in which a line meets a conic. [8]

3. Polar equation of straight lines and circles. Polar equation of a conic referred to a focus as pole. Equations of tangent, normal, chord of contact. [5]

Three Dimensions (15 Marks)

1. Rectangular Cartesian co-ordinates in space. Halves and Octants. Concept of a geometric vector (directed line segment). Projection of a vector on a co-ordinate axis. Inclination of a vector with an axis. Co-ordinates of a vector. Direction cosines of a vector. Distance between two points. Division of a directed line segment in a given ratio. [4]

2. Equation of Plane : General form, Intercept and Normal form. The sides of a plane. Signed distance of a point from a plane. Equation of a plane passing through the intersection of two planes.
    Angle between two intersecting planes. Bisectors of angles between two intersecting planes. Parallelism and perpendicularity of two planes.
    [8]

3. Straight lines in space : Equation (Symmetric & Parametric form). Direction ratio and Direction cosines. Canonical equation of the line of intersection of two intersecting planes. Angle between two lines. Distance of a point from a line. Condition of coplanarity of two lines. Equations of skew-lines. Shortest distance between two skew lines.
    [10]

1. Real numbers : Field axioms for numbers and other salient properties taken as axioms. Arithmetic continuum, well-ordering principle for N. Concept of ordered field. Concept of point set in one dimension. Bounded set. Least upper bound axiom or completeness axiom. Archimedean property and density property. Characterisation of R as a complete, Archimedean, ordered field and Q as Archimedean, ordered field, Symbols ∞ and $-\infty$. Symbols of intervals. [5]

2. Sequence of points in one dimension: Bounds, Limits, Convergence and non convergence, Operations on limits, Sandwich rule. Monotone sequences and their convergence. Nested interval theorem: Cauchy's general principle of convergence. Cauchy sequence. Limit of some important sequences with special reference to $\left\{\left(1+\dfrac{1}{n}\right)^{n}\right\}_{n}$. Cauchy's first and second limit theorems. [10]

3. Point set in one dimension: (a) Denumerable, at most denumerable and non denumerable sets. A sub-set of a denumerable set is either finite or denumerable. Union of (i) a finite set and a denumerable set (ii) two denumerable sets (iii) denumerable number of denumerable set. Denumerability of rational number. Non denumerability of points in a firite interval and of the set of all real numbers, (b) Neighbourhood of a point. Interior point. Accumulation point and isolated point of a linear point set. Bolzano-Weierstrass Theorem on accumulation point, Derived set, Open set and closed set. Union, Intersection, Complement of open and closed sets in R. No non-empty proper sub-set of R is both open and closed in R. Closure of a set to be defined as the union of the set and its derived set. Interior of a set, Deduction of basic properties of interior of a set and closure of a set. [10]

4. Real valued functions defined on intervals: Bounded functions. Step functions. Monotone functions. Composition of functions. Limits of functions: Algebra of limits and Sandwich rule. Cauchy criterion for the existence of finite limit. Important limits like $(\sin x)/x$, $\{\log(l+x)\}/x$, $(a^{x}-l)/x(a>0)$ as $x \to 0$. [5]

5. Continuity of a function at a point and on an interval: Neighbourhood properties, continuty of x^{n}, sin x, cos x, log x to be established.
   Continuity of composite function. Piecewise continuous functions. Discontinuity of function - type of discontinuity, ordinary discontinuity

of monotonic functions. [7]

Group - B (20 Marks)
Application of Differential Calculus and evaluation of integral I

1. Concept of a plane curve : Closed curve, simple curve.

   a) Tangents and Normals: Subtangent and sub-normals. Angle of intersection of curves. Pedal equation of a curve, Pedal of a curve

   b) Rectilinear asymptotes of a curve (Cartesian, Parametric and polar form).

   c) Curvature—Radius of curvature. Centre of curvature, Chord of curvature. Evolute of a curve.

   d) Concavity, convexity, singular points, nodes, cusps, points of inflexion—simple problems on species of cusps of a curve.

   e) Envelopes of one parameter and two parameter family of curves. Envelope as singular point locus-Evolute.

   v) Curve tracing-familiarity with well-known curves. [15]

2. Indefinite and suitable corresponding definite integrals for the functions. $1/(a + b \cos x)^n$, $(l \cos x + m \sin x)/ (p \cos x + q \sin x)$, $l/(x^2 + a^2)^n$. $\cos^m x$. $\sin^n x$, $\cos^m x \sin nx$ etc. where l, m, p, q, n are integers. Simple problems on definite integrals as the limit of a sum. [5]

3. Working knowledge of Beta and Gamma function (convergence to be assumed) and their interrelation (no proof). Use of the result $\Gamma(n)\Gamma(1-n) = \pi/\sin n\pi$ where 0<n<1. Computation of the integrals

   $\int_0^{\pi/2} \sin^n x\, dx, \int_0^{\pi/2} \cos^n x\, dx, \int_0^{\pi/2} \tan^n x\, dx, \int_0^{\pi/2} \sin^m x \cos^n x\, dx$ etc. when they exist,

   (using Beta function and Gamma function). [5]

Group-C (30 Marks)
(Differential Equation I)

1. Significance of ordinary differential equations. Geometrical and physical consideration. Formation of differential equation by elimination of arbitrary constant. Meaning of the solution of ordinary differential equation.
   Concept of linear and non-linear differential equations. [2]

2. Equations of first order and first degree: Statement of Existence theorem. Separable, Homogeneous and Exact equations. Condition of exactness, Integrating factor, Rules of finding integrating factor. (Statement of relevant results only) [5]

3. First order linear equations: Integrating factor (Statement of relevant results only). Equations reducible to first order linear equations. [2]

4. Equations of first order but not of first degree. Clairaut's equation. Singular solution. [3]

5. Applications : Geometric applications, Orthogonal trajectories. [2]

6. Higher order linear equations with constant co-efficients: Complementary Function, Particular Integral. Method of undetermined co-efficients, Symbolic operator D. Method of variation of parameters. Exact Equation.
   Euler's homogeneous equation and Reduction to an equation of constant co-efficients. [8]

7. Second order linear equations with variable co-efficients:

$$\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + Q(x)y = F(x).$$

   Reduction of order when one solution of the homogeneous part is known. Complete solution. Method of variation of parameters. Reduction to Normal form. Change of independent variable. Operational Factors. [10]

Group-D (25 Marks)
(Vector Algebra and Analysis-I)

1. Vector Algebra: Vector (directed line segment) Equality of two free vectors, Addition of Vectors, Multiplication by a Scalar.
    Position vector, Point of division, Conditions of collinearity of three points and co-planarity of four points.
    Rectangular components of a vector in two and three dimensions.
    Product of two or more vectors. Scalar and vector products, Scalar triple products and Vector triple products. Product of four vectors.
    Direct application of Vector Algebra in (i) Geometrical and Trigonometrical problems (ii) Work done by a force, Moment of a force about a point.
    Vector equations of straight lines and planes. Volume of a tetrahedron. Shortest distance between two skew lines.   [15]

2. Vector differentiation with respect to a scalar variable, Vector functions of one scalar variable, Derivative of a vector. Second derivative of a vector. Derivatives of sums and products. Velocity and Acceleration as derivative.   [5]

3. Concepts of scalar and vector fields Direction derivative, Gradient, Divergence and curl, Laplacian.   [5]

PART-II

Paper-III

Group-A (15 Marks)
(Classical Algebra II)

1. Polynomial equations with real co-efficients: Reciprocal equations. Cardan's method of solving a cubic equation. Ferrari's method of solving a biquadratic equation. Binomial equation. Special roots.   [7]

2. Inequalities AM $\geq$ GM $\geq$ HM and their generalisations : the theorem of weighted means and m th. power theorem. Cauchy's inequality (statement only) and its direct applications.   [8]

Group-B (10 Marks)
(Modern Algebra II)

1. Cosets and Lagrange's theorem on finite group. Cyclic group : Definition and examples. Sub-groups of a cyclic group, Generator. Necessary and sufficient condition for a finite group to be cyclic. Permutations : Cycle, Transposition. Every $\sigma \, \varepsilon \, S_n$ (Symbols have their usual meanings) can be expressed as the product of disjoint cycles. Even and odd permutations. Permutation Group, Symmetric group, Alternating group: Order of an alternating group. [10]

Group-C (20 Marks)
(Linear Algebra II)

1. Vector/Linear space over a field with special reference to spaces of n tuples of real numbers. Examples of vector space from different branches of Mathematics. Sub-spaces. Union and intersection of vector sub- spaces. Sum of two sub-spaces.

    Linear combinations. Linear dependence and independence of a finite set of vectors. Linear span. Generators of a vector space. Finite dimensional vector space.

    Existence of Basis, Replacement Theorem. Any two bases have the same number of basis vectors. Extension theorem - Extraction of basis from generators. Formation of basis from linearly independent sub-set. Special emphasis on R^n. Examples from R^n ($n \leq 4$). [10]

2. Row Space and Column Space of a Matrix. Definition of row space and column space of a matrix, Row rank, Column rank. Rank of a matrix. Rank (AB) $\leq$ Min (Rank A, Rank B). [6]

3. Linear homogeneous system of equations : Solution space as a sub-space. For a homogeneous system AX = 0 in n unknowns, Rank X (A) + Rank A = n. The homogeneous system AX = 0 containing n equations in n unknowns has a non-trivial solution iff Rank A < n. System of linear non-homogeneous equations : Necessary and sufficient condition for the consistency of the system. Solution of the system of equations (Matrix method, Cramer's Rule). [5]

4. Characteristic equation of a square matrix. Eigen value and Eigen vector. Cayley-Hamilton Theorem. Simple properties of Eigen value and Eigen vector. Diagonalisation of matrices. [6]

5. Inner Product Space : Definition and examples. Norm. Euclidean vector spaces (EVS), Triangle Inequality and Cauchy-Schwarz Inequality in

EVS. Orthogonality of vectors. Orthonormal basis, Gram-Schmidt process of orthonormalisation. [8]

Group-D (20 Marks)
(Real Analysis II)

1. Sub-sequence : All the sub-sequences of a convergent sequence are convergent and converge to the same limit as that of original sequence. Every bounded sequence has a convergent sub-sequence. Subsequential limits. Upper limit and Lower limit as the L.U.B. and G.L.B. respectively of a set containing all the subsequential limits-other equivalent definitions. Inequalities and equalities with upper and lower limits. A sequence is convergent iff its upper and lower limits be equal. [10]

2. Infinite series of real numbers : Convergence, divergence Cauchy's criterion of convergence. Abel-Pringsheim's Test. Series of non-negative real numbers: Tests of convergence-Cauchy's condensation test. Upper limit and lower limit criteria for (i) Comparison test, (ii) Ratio test, (iii) Root test, (iv) Rummer's test. Statements of Raabe's test, Bertrand's test, Logarithmic test and Gauss test.

   Series of arbitrary terms : Absolutely convergent and conditionally convergent series. Alternating series : Leibnitz test, Root test and Ratio test. Non-absolute convergence-Abel's and Dirichlet's test (statements and applications).

   Rearrangement of series through examples. Riemann's re-arrangement theorem (statement) and simple examples. Rearrangement of absolutely convergent series. [10]

3. Real valued functions defined on an interval : Uniform continuity. Properties of continuous functions on closed intervals : Boundedness, attainment of bounds, Bolzano's theorem. Intermediate-value property and allied results.

   Continuous function carries closed and bounded interval into closed and bounded interval. Functions continuous on a closed and bounded interval I is uniformly continuous on I. A necessary and sufficient condition under which a continuous function on a bounded open interval I will be uniformly continuous on I. Lipschitz condition and uniform continuity. Existence of inverse function of a strictly monotone function and its continuity with special reference to inverse circular functions. [15]

4. Concept of differentiability and differential : chain rule, sign of derivative. For a differentiable function Lipschitz condition is equivalent to boundedness of the derivative. Successive derivative : Leibnitz theorem. Theorems on derivatives : Darboux theorem, Rolle's theorem,

Mean value theorem of Lagrange and Cauchy, Taylor's theorem with Schlomilch-Rouche's form of remainder, Lagrange's and Cauchy's form of remainder. Young's form of Taylor's theorem. Maclaurin's series. Expansion of e^x, a^x ($a > 0$), log $(1 + x)$, $(1 + x)^m$, sin x, cos x etc. with their ranges of validity. [10]

5. Indeterminate forms: Statement of L. Hospital's rule and its consequences. [2]

6. Point of local extremum (maximum, minimum and saddle point) of a function in an interval. Sufficient condition for the existence of a local maximum/ minimum of a function at a point. Application of the principle of maximum/minimum in geometrical and physical problems. [5]

Group-E (25 Marks)
(Functions of Several Variables)

1. Point set in two and three dimensions - Concept only of neighbourhood of a point, interior point, accumulation point, open set, closed set, Bolzano-Weierstrass theorem (statement only) in R^2. [2]

2. Concept (only) of R^n and examples of functions on R^n. [1]

3. a) Functions of two and three variables - Limit and continuity, Partial derivatives. Sufficient condition for continuity. Relevant results regarding repeated limits and double limits.

   b) Functions $R^2 \rightarrow R^1$ - Differentiability and its sufficient condition, differential as a map, Chain rule. Euler's theorem and its converse. Commutativity of the order of partial derivatives - Theorem of Young and Schwarz. [12]

4. Jacobian for functions of two and three variables - Simple properties including functional dependence. Concept of Implicit Function : Statement and simple application of implicit function theorem for two variables. Differentiation of implicit function. Jacobian of implicit function. Partial derivative as ratio of two Jacobians in case of function of two variables. [10]

Group-F (10 Marks)
(Application of Integral Calculus II)

1. Area : Area enclosed by a curve, area enclosed between a curve and a secant, area between two curves and area between a curve and its asymptote (if there be any). [2]

2. Problems on volume and surface area of solids of revolution. Statement of Pappus theorem and its direct application to well-known curves. [2]

3. Determination of C. G. and moments & products of inertia-simple problems only. [3]

Paper-IV
Group-A (20 Marks)
(Analytical Geometry of Two & Three dimensions-II)

1. Circle, Parabola, Ellipse and Hyperbola : Equations of pair of tangents, from an external point, chord of contact, poles and polars, conjugate points and conjugate lines. [4]

2. Sphere (General Equation, Circle, Sphere through the intersection of two spheres, Radical Plane, Tangent, Normal).
    Cone (Right circular cone, General homogeneous second degree equation. Section of a cone by a plane as a conic and as a pair of lines, Condition for three perpendicular generators, Reciprocal cone, Enveloping cone). Cylinder (Generators parallel to either of the axes, general form of equation. Right-circular cylinder, Enveloping cylinder).
    Surface of Revolution (about axes of reference only). Ruled surface. [15]

3. Transformation of rectangular axes by translation, rotation and their combinations. [2]

4. General equation of second degree in three variables: Reduction to canonical forms. Classification of Quadrics. [2]

5. Ellipsoid, Hyperboloid, Paraboloid : Canonical equations and the study of their shape. [5]

6. Tangent planes, Normals, Enveloping cone. [5]

7. Generating lines of hyperboloid of one sheet and hyperbolic paraboloid. [8]

8. Knowledge of Cylindrical, Polar and Spherical polar co-ordinates, their relations (No deduction required). [2]

Group-B (10 Marks)
(Differential Equation II)

1. Simple eigenvalue problems. [2]

2. Simultaneous linear differential equations. Total differential equation: Condition of integrability. [3]

3. Partial differential equation (PDE) : Introduction, Formation of P.D.E, Solution of PDE by Lagrange's method of solution and by Charpit's method. [5]

Group-C (25 Marks)
(Linear Programming and Game Theory)

1. Hyperplane, Convex set, Cone, Extreme points, convex hull and convex polyhedron, Supporting and Separating hyperplane.

   The collection of all feasible solutions of an L.P.P. constitutes a convex set. The extreme points of the convex set of feasible solutions correspond to its B.F.S. and conversely. The objective function has its optimal value at an extreme point of the convex polyhedron generated by the set of feasible solutions, (the convex polyhedron may also be unbounded). In the absence of degeneracy, if the L.P.P. admits of an optimal solution, then at least one B.F.S. must be optimal. Reduction of a F.S. to a B.F.S. [6]

2. Slack and surplus variables. Standard form of L.P.P. Theory of simplex method. Feasibility and optimality conditions. [6]

3. The algorithm. Two phase method, Degeneracy in L.P.P. and its resolution. [8]

4. Duality Theory. The dual of the dual is the primal. Relation between the objective values of dual and the primal problems. Relation between their optimal values. Complementary slackness, Duality and simplex method and their applications. [6]

5. Transportation and Assignment problems. Mathematical justification for optimality criterion. Hungarian method. Travelling Salesman problem.[8]

6. Concept of Game problem. Rectangular games. Pure strategy and Mixed strategy. Saddle point and its existence. Optimal strategy and value of the game. Necessary and sufficient condition for a given strategy to be optimal in a game. Concept of Dominance. Fundamental Theorem of Rectangular games. Algebraic method. Graphical method and Dominance method of solving Rectangular games. Inter-relation

between the theory of Games and L.P.P.   [10]

Group-D (45 Marks)
(Analytical Dynamics of a Particle)
(Acquaintance with elementary concepts of Statics is assumed)

1. Fundamental Ideas and Principles of Dynamics. Laws of motion. Work, Power and Energy. Principles of conservation of energy and of momentum - Impulse and Impulsive forces.   [5]

2. Motion in a straight line under variable acceleration. Motion under inverse square law, Composition of two S. H. M's of nearly equal frequencies. Motion of a particle tied to one end of an elastic string. Rectilinear motion in a resisting medium. Damped forced oscillation. Motion under gravity where the resistance varies as some integral power of velocity, Terminal velocity.   [10]

3. Impact of elastic bodies. Newton's experimental law of elastic impact. Direct impact. Loss of K.E. in a direct impact Oblique impact of two elastic spheres, Loss of K. E. in oblique impact. Angle of deflection. [3]

4. Expressions for velocity and acceleration of a particle moving on a plane in Cartesian and polar co-ordinates. Motion of particle moving in a plane with reference to a set of rotating axes. Motion of a particle in plane.   [6]

5. Central forces and central orbits. Characteristics of central orbits. Stability of nearly circular orbits.   [6]

6. Tangential and Normal accelerations. Circular motion. Motion of a train or cyclist on a banked tract. Simple cases of constrained motion of a particle.   [4]

7. Motion of a particle in a plane under different laws of resistance. Motion of a projectile in a resisting medium in which the resistance varies as the velocity. Trajectories in a resisting medium where resistance varies as some integral power of the velocity.   [5]

8. Motion on a smooth curve under resistance.   [2]

9. Motion under inverse square law in a plane. Escape velocity, Planetary motion and Kepler's laws. Time of describing an arc of the orbit. Motion of artificial satellite Slightly disturbed orbits.   [6]

10. Conservative field of force and principle of conservation of energy. Motion of a rough curve (such as circle, parabola, ellipse, cycloid etc.)

under gravity. [6]

11. Equation of motion of a particle of varying mass. Simple problems of motion of varying mass such as those of falling raindrops and projected rockets. [6]

ANNEXURE : LIST OF BOOKS FOR REFERENCE

Paper-I, Group A & Paper-III, Group A

1. The Theory of Equations (Vol. I) - Burnside and Panton.
2. Higher Algebra - Barnard and Child.
3. Higher Algebra - Kurosh (Mir)

Paper-I, Group B & Paper-III, Group B

1. Modern Algebra - Surjeet Singh & Zameruddin.
2. First Course in Abstract Algebra – Fraleigh
3. Topics in Algebra – Hernstein
4. Text book of algebra - Leadership Project Committee (University of Bombay)
5. Elements of Abstract Algebra - Sharma, Gokhroo, Saini (Jaipur Publishing House, S. M. S. Highway, Jaipur-3).
6. Abstract Algebra - N. P. Chaudhuri (Tata Mc.Graw Hill).

Paper-I, Group C & Paper-III, Group C

1. Linear Algebra - Hadley
2. Text Book of Matrix - B. S. Vaatsa

Paper-I, Group D & Paper IV, Group A

1. Co-ordinate Geometry - S. L. Loney
2. Co-ordinate Geometry of Three Dimensions - Robert J. T. Bell
3. Elementary Treatise on Conic sections —. C. Smith
4. Solid Analytic Geometry - C. Smith.

Paper-II, Group A & Paper-III, Group D, Group E

1. Basic Real & Abstract Analysis - Randolph J. P. (Academic Press)
2. A First Course in Real Analysis - M. H. Protter & G. B. Morrey (Springer Verlag, NBHM)
3. A Course of Analysis - Phillips.
4. Problems in Mathematical Analysis - B. P. Demidovich (Mir)
5. Problems in Mathematical Analysis - Berman (Mir)
6. Differential & Integral Calculus   (Vols. I & II) - Courant & John.

7. Calculus of One Variable-Maron (CBS Publication)

8. Introduction to Real Analysis - Bartle & Sherbert (John Wiley & sons)

9. Mathematical Analysis - Parzynski.

10. Introduction to Real Variable theory - Saxena & Shah (Prentice Hall Publication)

11. Real Analysis - Ravi Prakash & Siri Wasan (Tata McGraw Hill)

12. Mathematical Analysis - Shantinarayan (S. Chand & Co.)

13. Theory & Applications of Infinite Series - Dr. K. Knopp

14. Advanced Calculus - David Widder (Prentice Hall)

Paper-II, Group B *&* Paper-III, Group F

1. Differential Calculus - Shantinaryan.

2. Intergral Calculus – Shantinarayan

3. An elementary treatise on the Differential Calculus - J. Edwards (Radha Publishing House)

Paper-II, Group D

1. Vector Analysis - Louis Brand.

2. Vector Analysis - Barry Spain

3. Vector & Tensor Analysis - Spiegel (Schaum)

4. Elementary Ventor Analysis - C. E. Weatherburn (Vol. I & II)

Paper-I, Group C & Paper-IV Group C

1. Linear Programming : Method and application - S. I. Gass

2. Linear Programming - G. Hadley

3. An Introduction to Linear Programming & Theory of Games - S. Vajda.

Paper-II, Group C & Paper IV, Group B

1. Differential Equations - Lester R. Ford (McGraw Hill)

2. Differential Equations - S. L. Ross (John Wiley)

3. Differential Equations - H. T. H. Piaggio

4. A Text book of Ordinary Differential Equations - Kiseleyev, Makarenko & Krasnov (Mir)

5. Differential Equations - H. B. Phillips (John Wiley & Sons)

6. Differential Equations with Application & Programs - S. Balachanda Rao, H. R. Anuradha (University Press)

7. Text Book of Ordinary Differential Equations (2nd Ed.) - S. G. Deo, V. Lakshmikantham & V. Raghavendra (Tata McGraw Hill).

8. An Elementary Course in Partial Differential Equation-T. Amarnath

(Narosa)

9. An Introductory Course on Ordinary Differential Equation-D.A. Murray.

Paper IV, Group D:

1. An Elementary Treatise on the Dynamics of a Particle & of Rigid bodies
- S. L. Loney (Macmillan)

<center>

PART-III
Paper-V
Group-A (70 Marks)
(Real Analysis-II)

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1. *Linear Point Set:* Covering by open intervals. Sub-covering. Cantor intersection theorem.' Lindelof-covering theorem (statement only). Compact sets. Heine-Borel Theorem and its converse. [5]

2. *Functions defined on point sets in one dimension:* Limit and continuity. Continuity on compact set. Uniform continuity on compact set. Inverse function. Continuous image of compact set is compact. [5]

3. a) Sequence of functions defined on a set ($\subset R$): Pointwise and uniform convergence. Cauchy criterion of uniform convergence. Dini's theorem on uniform convergence. Weierstrass' M-test.
Limit function: Boundedness. Repeated limits. Continuity. Integrability and differentiability of the limit function of a sequence of functions in case of uniform convergence.

   b) *Series of functions defined on a set:* Pointwise and uniform convergence. Cauchy criterion of uniform convergence. Dini's theorem on uniform convergence. Tests of uniform convergence -Weierstrass' M-test. Statement of Abel's and Dirichlet's test and their applications. Passage to the limit term by term. Sum function : boundedness, continuity, integrability, differentiability of a series of functions in case of uniform convergence.

   c) *Power Series (P.S.):* Fundamental theorem of Power Series. Cauchy-Hadamard theorem. Determination of radius of convergence. Uniform and absolute convergence of P.S. Properties of sum function. Abel's limit theorems. Uniqueness of power series having same sum function.
   Exponential, logarithmic and trigonometric functions defined by Power Series and deduction of their salient properties. [20]

4. a) *Function of two variables:* $f: R^2 \rightarrow R^1$. Mean value theorem and Taylor's theorem.

   b) Extremum of functions of two and three variables: Lagrange's Method of undetermined multipliers. [3]

5. *Riemann Integration for bounded functions:* Partition and

<center>**20**</center>

refinement of partition of an interval. Upper Darboux sum U(P, f) & Lower Darboux sum L (P, f) and associated results. Upper Riemann (Darboux) integral and Lower Riemann (Darboux) integral. Darboux's theorem. Necessary and sufficient condition of R-integrability.

Classes of Riemann Integrable functions: Monotone functions, continuous functions, piecewise continuous functions with (i) finite number of points of discontinuities, (ii) infinite number of points of discontinuities having finite number of accumulation points.

Riemann Sum : Alternative definition of integrability. Equivalence of two definitions (statement only).

Integrability of sum, product, quotient, modulus of R-integrable functions.

Sufficient condition for integrability of composition of R-. integrable functions.

Properties of Riemann integrable functions arising from the above results.

Function defined by definite integral $\int_a^x f(t)dt$ and its properties.

Primitive or Indefinite Integral. Properties of definite integral. Definition of log *x (x > 0)* as an integral and deduction of simple properties including its range. Definition of *e* and its simple properties. Fundamental theorem of Integral Calculus. First Mean Value Theorem of Integral Calculus. Statements and applications of Second Mean Value Theorem of Integral Calculus (both Bonnett's form and Weierstrass form) Theorem on method of substitution for continuous functions.   [10]

6. *Improper Integral:* Range of integration, finite or infinite. Necessary and sufficient condition for convergence of Improper Integral in both cases.

Tests of convergence: Comparison and μ-Test. Absolute and non-absolute convergence - Corresponding Tests. Beta and Gamma functions - their convergence and inter-relations. Statement of Abel's and Dirichlet's Tests for convergence of the integral of a product.

Uniform convergence of Improper Integral by M-Test.

$$\left[\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin nx}, 0 < n < 1, \text{ to be assumed} \right]$$ [8]

7. *Definite Integral as a function of a parameter:* Differentiation and Integration with respect to the parameter under integral sign –

Statements (only) of some relevant theorems and simple problems. [3]

8. i) *Concept on function of Bounded Variation (BV):* Monotonic function is of BV. If *f* be of BV on *[a, b]*, then f is bounded on *[a. b].* Examples of functions of BV which are not continuous and continuous functions not of BV. Statement of a necessary and sufficient condition for a function *f* to* be of BV on *[a, b]* is that *f can* be written as the difference of two monotonic increasing functions on *[a, b]* [3]

   ii) *Rectification of Plane Curves:* Definition of Rectifiable Curve. A plane curve *v = (f, g)* is rectifiable if and only if *f* and g be both of bounded variation (Statement only). Simple examples on determination of length of curves.

   Determinations of intrinsic equation of a curve. [5]

9. *Fourier Series:* Trigonometric Series. Fourier co-efficients. A periodic function of bounded variation can be expressed as a Fourier series (Statement only). Statement of Dirichlet's conditions of convergence. Half-range series, sine and cosine series. [5]

10. i) *Double Integral:* Concept of Upper sum, Lower sum, Upper Integral, Lower Integral and Double Integral (no rigorous treatment is needed). Statement of Existence Theorem for continuous functions. Change of order of integration. Triple integral. Transformation of double and triple Integrals (Problems only).

   ii) *Determination* of volume and surface area by Multiple Integrals (Problems only). [5]

Group-B (15 Marks)
(Metric Space)

Definition and examples of Metric Space. Neighbourhoods. Limit points. Interior points. Open and closed sets. Closure and Interior. Boundary points. Sub-space of a Metric Space. Cauchy Sequences. Completeness. Cantor Intersection Theorem. Construction of real number as the completion of the incomplete metric spaces of rationals. Real number as a complete ordered field (No proof of theorem). [10]

Complex Analysis

Complex numbers as ordered pairs. Geometric representation of complex numbers. Stereographic projection.

Complex functions: Continuity and differentiability of complex functions. Analytic functions. Cauchy-Riemann Equations. Statement of Milne's Method, Harmonic functions. [10]

Paper-VI
Group-A (50 Marks)
(Probability and Statistics)

Mathematical Theory of Probability:

Random experiments. Simple and compound events. Event space. Classical and frequency definitions of probability and their drawbacks. Axioms of Probability. Statistical regularity. Multiplication rule of probabilities. Bayes' theorem. Independent events. Independent random experiments. Independent trials. Bernouli trials and binomial law. Poisson trials. Random variables. Probability distribution. Distribution function. Discrete and continuous distributions. Binomial, Poisson, Gamma, Uniform and Normal distribution. Poisson Process (only definition). Transformation of random variables. Two dimensional probability distributions. Discrete and continuous distributions in two dimensions. Uniform distribution and two dimensional normal distribution, conditional distributions. Transformation of random variables in two dimensions. Mathematical expectation. Mean, variance, moments, central moments. Measures of location, dispersion, skewness and kurtosis. Median, mode, quartiles. Moment-generating function. Characteristic function. Two-dimensional expectation. Covariance, Correlation co-efficient, Joint characteristic function. Multiplication rule for expectations. Conditional expectation. Regression curves, least square regression lines and parabolas. Chi-square and t-distributions and their important properties (Statements only) Tchebycheff's inequality. Convergenece in probability. Statements of : Bernoulli's limit theorem, Law of large numbers, Poisson's approximation to binomial distribution and normal approximation to binomial distribution. Concepts of asymptotically normal distribution. Statement of central limit theorem in the case of equal components and of limit theorem for characteristic functions (Stress should be more on the distribution function

theory than on combinatorial problems. Difficult combinatorial problems should be avoided). [40]

Mathematical Statistics:

Random sample. Concept of sampling and various types of sampling. Sample and population. Collection, tabulation and graphical representation. Grouping of data, Sample characteristic and their computation. Sampling distribution of a statistic. Estimates of a population characteristic or parameter. Unbiased and consistent estimates. Sample characteristics as estimates of the corresponding population characteristics. Sampling distributions of the sample mean and variance. Exact sampling distributions for the normal populations. Bivariate samples. Scatter diagram. Sample correlation co-efficient. Least square regression lines and parabolas. Estimation of parameters. Method of maximum likelihood. Applications to binomial, Poisson and normal population. Confidence intervals. Interval estimation for parameters of normal population. Statistical hypothesis. Simple and composite hypothesis. Best critical region of a test. Neyman-Pearson theorem (Statement only) and its application to normal population. Likelihood ratio testing and its application to normal population. Simple applications of hypothesis testing (for practical). [35]

Group-B (50 Marks)
(Numerical Analysis and Computer Programming)

Numerical Analysis:

What is Numerical Analysis?

Errors in Numerical computation: Gross error, Round off error, Truncation error. Approximate numbers. Significant figures. Absolute, relative and percentage error.

Operators : $\Delta, \nabla, E, \mu, \delta$ (Definitions and simple relations among them)

Interpolation : Problems of interpolation, Weierstrass' approximation theorem (only statement). Polynomial interpolation. Equispaced arguments. Difference table. Deduction of Newton's forward and backward interpolation formulae. Statements of Stirling's and Bessel's interpolation formulae. Error terms. General interpolation formulae : Deduction of Lagrange's interpolation formula. Divided difference. Newton's General Interpolation formula (only statement). Inverse interpolation.

Interpolation formulae using the values of both f(x) and its derivative *f(x) :* Idea of Hermite interpolation formula (only the basic concepts).

Numerical Differentiation based on Newton's forward & backward and Lagrange's formulae.

Numerical Integration : Integration of Newton's interpolation formula.

Newton - Cote's formula. Basic Trapezoidal and Simpson's 1/3 rd. formulae. Their composite forms. Weddle's rule (only statement). Statement of the error terms associated with these formulae. Degree of precision (only definition).

Numerical solution of non-linear equations : Location of a real root by tabular method. Bisection method. Secant/Regula-Falsi and Newton-Raphson methods, their geometrical significance. Fixed point iteration method.

Numerical solution of a system of linear equations: Gauss elimination method. Iterative method - Gauss-Seidel method. Matrix inversion by Gauss elimination method (only problems - up to 3x3 order).

Eigenvalue Problems : Power method for numerically extreme eigenvalues.

Numerical solution of Ordinary Differential Equation : Basic ideas, nature of the problem. Picard, Euler and Runge-Kutta (4th order) methods (emphasis on the problems only).   [30]

Fundamentals of Computer Science and Computer Programming:

Computer fundamentals : Historical evolution, computer generations, functional description, operating systems, hardward & software.

Positional number systems : binary, octal', decimal, hexadecimal systems. Binary arithmetic.

Storing of data in a computer : BIT, BYTE, Word. Coding of data - ASCIL, EBCDIC, etc.

Algorithm and Flow Chart : Important features. Ideas about the complexities of algorithm. Application in simple problems.

Programming languages : General concepts, Machine language, Assembly Language, High Level Languages. Compiler and Interpreter. Object and Source Program. Ideas about some major HLL.

Students are required to opt for any one of the following two programming languages :

(1)   Programming with FORTRAN 77/90

or   (2)   Introduction to ANSI C :

Programming with FORTRAN 77/90 :

Introduction, Keywords, Constants and Variables-integer, real, complex, logical, character, double precision, subscripted. Fortran expressions. I/O statements-formatted and unformatted. Program execution control-logical if, if-then-else, etc. Arrays-Dimension statement. Repetitive computations-Do, Nested Do, etc. Sub-programs : Function sub program and Subroutine sub program.

Application   to   simple   problems : Evaluation of functional

values, solution of quadratic equations, approximate sum of convergent infinite series, sorting of real numbers, numerical integration, numerical solution of non-linear equations, numerical solution of ordinary differential equations, etc.

Introduction to ANSI C:

Character set in ANSI C. Key words: if, while, do, for, int, char, float etc.

Data type : character, integer, floating point, etc. Variables, Operators :=, = =, !!<,>, etc. (arithmetic, assignment, relational, logical, increment, etc.).Expressions : e.g. (a = = b) !! (b = = c), Statements : e.g. if (a < b) small= a; else small = b. Standard input/output. Use of while, if .... else, for, do... while, switch, continue, etc. Arrays, strings, Function definition. Running simple C Programs. Header File. [30]

Books for Reference

1. The elements of probability theory and some of its applications : H. Cramer
2. An introduction to probability theory and its applications (Vol 1) : W. Feller
3. Mathematical methods of statistics : H. Cramer
4. Theory of probability : B. V. Gnedenko
5. Mathematical probability : J. V. Uspensky
6. Structured FORTRAN 77 for engineers and scientists : D. M. Etter (The Benjamin/Cummings Publishing Co. Inc.)
7. Programming with FORTRAN 77-A structured approach : R. S. Dhaliwal, S. K. Agarwal, S. K. Gupta (Wiley Eastern Limited/New Age International Ltd.)
8. Programming and computing with FORTRAN 77/90 : P. S. Grover (Allied Publishers)
9. Programming with FORTRAN including structured FORTRAN : Seymour Lipschutz and Arthur Poe (Schaum's Outline Series)
10. FORTRAN 77 and numerical methods : C. Xavier (Wiley Eastern limited)
11. Numerical methods: E. Balagurusamy (Tata McGraw-Hill Publishing Co.)
12. Let us C : Yashvant Kanetkar (BPB Publications)
13. Programming in C : V. Krishnamoorthy and K. R. Radhakrishnan (Tata McGraw Hill).
14. C by example : Noel Kalicharan (Cambridge University Press)

15. Programming in ANSI C: E. Balagurusamy (Tata McGraw Hill).

16. Introduction to numerical analysis : F. B. Hilderbrand (TMH Edition)

17. Numerical Analysis : J. Scarborough

18. Introduction to numerical analysis : Carl Erik Froberg (Addison Wesley Publishing)

19. Numerical methods for science and engineering : R. G. Stanton (Prentice Hall)

<div style="background:orange">Paper-VII</div>
Group-A (10 Marks)
(Vector Analysis II)

Line integrals as integrals of vectors, circulation, irrotational vector, work done, conservative force, potential orientation. Statements and verification of Green's theorem, Stokes' theorem and Divergence theorem.　　　　[8]

Group-B (55 Marks)
(Analytical Statics)

1. Friction : Laws of Friction, Angle of friction, Cone of friction. To find the positions of equilibrium of a particle lying on a (i) rough plane curve, (ii) rough surface under the action of any given forces.　　　　[4]

2. Centre of Gravity : General formula for the determination of C.G. Determination of position of C. G. of any arc, area of solid of known shape by method of integration.　　　　[3]

3. Astatic Equilibrium, Astatic Centre. Positions of equilibrium of a particle lying on a smooth plane curve under action of given force. Action at a joint in a frame work.　　　　[4]

4. Virtual work : Principle of virtual work for a single particle. Deduction of the conditions of equilibrium of a particle under coplanar forces from the principle of virtual work. The principle of virtual work for a rigid body. Forces which do not appear in the equation of virtual work. Forces which appear in the equation of virtual work. The principle of virtual work for any system of coplanar forces acting on a rigid body. Converse of the principle of virtual work.　　　　[8]

5. Stable and Unstable equilibrium. Co-ordinates of a body and of a system of bodies. Field of forces. Conservative field. Potential energy of a system. The energy test of stability. Condition of stability of equilibrium of a perfectly rough heavy body lying on fixed body. Rocking stones.　　　　[6]

6. Forces in three dimensions. Moment of a force about a line. Axis

of a couple. Resultant of any two couples acting on a body. Resultant of any number of couples acting on a rigid body. Reduction of a system of forces acting on a rigid body. Resultant force is an invariant of the system but the resultant couple is not an invariant.

Conditions of equilibrium of a system of forces acting on a body. Deductions of the conditions of equilibrium of a system of forces acting on a rigid body from the principle of virtual work. Poinsot's central axis. A given system of forces can have only one central axis. Wrench, Pitch, Intensity and Screw. Condition that a given system of forces may have a single resultant. Invariants of a given system of forces. Equation of the central axis of a given system of forces.   [12]

Group-C (30 Marks)
(Rigid Dynamics)

Momental ellipsoid. Equimomental system. Principal axis. D'Alembert's principle. D'Alembert's equations of motion. Principles of moments. Principles of conservations of linear and angular momentum. Independence of the motion of centre of inertia and the motion relative to the centre of inertia. Principle of energy. Principle of conservation of energy.

Equation of motion of a rigid body about a fixed axis. Expression for kinetic energy and moment of momentum of a rigid body moving about a fixed axis. Compound pendulum. Interchangeability of the points of a suspension and centre of oscillation. Minimum time of oscillation. Reaction of axis of rotation.

Equations of motion of a rigid body moving in two dimensions. Expression for kinetic energy and angular momentum about the origin of rigid body moving in two dimensions. Two dimensional motion of a solid of revolution down a rough inclined plane. Necessary and sufficient condition for pure rolling. Two dimensional motion of a solid of revolution moving on a rough horizontal plane.

Equations of motion under impulsive forces. Equation of motion about a fixed axis under impulsive forces. Centre of percussion. To show that (i) if there is a definite straight line such that the sum of the moments of the external impulses acting on a system of particles about it vanishes, then the total angular momentum of the system about that line remains unaltered, (ii) the change of K. E. of a system of particles moving in any manner under the application of impulsive forces is equal to the work done by the impulsive forces. Impulsive forces applied to a rigid body moving in two dimensions.

Group-D (25 Marks)
(Hydrostatics)

1.   Definition of Fluid, Perfect Fluid, Pressure. To prove that the pressure at a point in a fluid in equilibrium is the same in every direction. Transmissibility of liquid pressure. Pressure of heavy fluids. To prove

   i)   In a fluid at rest under gravity the pressure is the same at all points in the same horizontal plane.

   ii) In a homogeneous fluid at rest under gravity the difference between the pressures at two points is proportional to the difference of their depths.

   iii) In a fluid at rest under gravity horizontal planes are surfaces of equal density.

   iv) When two fluids of different densities at rest under gravity do not mix, their surface of separation is a horizontal plane.
       Pressure in heavy homogeneous liquid. Thrust of heavy homogeneous liquid on plane surfaces.

2.   Definition of centre of pressure. Formula for the depth of the centre of pressure of a plane area. Position of the centre of pressure. Centre of pressure of a triangular area whose angular points are at different depths. Centre of pressure of a circular area. Position of the centre of pressure referred to co-ordinate axes through the centroid of the area. Centre of pressure of an elliptical area when its major axis is vertical or along the line of greatest slope. Effect of additional depth on centre of pressure.

3.   Equilibrium of fluids in given fields of force : Definition of field of force, line of force. Pressure derivative in terms of force. Surface of equi-pressure. To find the necessary and sufficient conditions of equilibrium of a fluid under the action of a force whose components are X, Y, Z along the co-ordinate axes. To prove (i) that surfaces of equal pressure are the surfaces intersecting orthogonally the lines of force , (ii) when the force system is conservative, the surfaces of equal pressure are equi-potential surfaces and are also surfaces of equal density. To find the differential equations of the surfaces of equal pressure and density.

4.   Rotating fluids. To determine the pressure at any point and the surfaces of equal pressure when a mass of homogeneous liquid

contained in a vessel, revolves uniformly about a vertical axis.

5. The stability of the equilibrium of floating bodies. Definition, stability of equilibrium of a floating body, metacentre, plane of floatation, surface of buoyancy. General propositions about small rotational displacements. To derive the condition for stability.

6. Pressure of gases. The atmosphere. Relation between pressure, density and temperature. Pressure in an isothermal atmosphere. Atmosphere in convective equilibrium. [30]

Books for Reference

1. Vector Analysis - Spiegel (Schaum)
2. Vector Calculus - C. E. Weatherburn
3. Analytical Statics - S. L. Loney
4. Dynamics of a Particle and of Rigid bodies - S. L. Loney.
5. Hydrostatics - A. S. Ramsay

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Paper-VIII A
Group-A (25 Marks)
(Algebra II)

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Section - 1: Linear Algebra (10 Marks)

1. Linear Transformation (L.T.) on Vector Spaces : Definition of L. T., Null space, range space of an L. T., Rank and Nullity, Sylvester's Law of Nullity. [Rank (T) + Nullity (T) = dim (V)]. Determination of rank (T), Nullity (T) of linear transformation $T : R^n \to R^m$. Inverse of Linear Transformation. Non-singular Linear Transformation.
    Change of basis by Linear Transformation. Vector spaces of Linear Transformations.

2. Linear Transformation and Matrices : Matrix of a linear transformation relative to ordered bases of finite-dimensional vector spaces. Correspondence between Linear Transformations and Matrices. Linear Transformation is non-singular if its representative matrix be non-singular. Rank of L.T. = Rank of the corresponding matrix. [5]

Section-2 : Modern Algebra (8 Marks)

3. Normal sub-groups of a Group : Definition and examples. Intersection, union of normal sub-groups. Product of a normal sub-group and a sub

group. Quotient Group of a Group by a normal sub-group. [5]

4. Homomorphism and Isomorphism of Groups. Kernel of a Homomorphism. First Isomorphism Theorem. Properties deducible from definition of morphism. An infinite cyclic group is isomorphic to (Z, +) and a finite cyclic group of order *n* is isomorphic to the group of residue classes modulo *n*. [5]

Section - 3 : Boolean Algebra (7 Marks)

5. Boolean Algebra : Huntington Postulates for Boolean Algebra. Algebra of sets and Switching Algebra as examples of Boolean Algebra. Statement of principle of duality. Disjunctive normal and Conjunctive normal forms of Boolean Expressions. Design of simple switching circuits. [10]

Group-B (15 Marks)
(Differential Equations III)

1. Laplace Transform and its application in ordinary differential equations: Laplace Transform and Inverse Laplace Transform. Statement of Existence theorem. Elementary properties of Laplace Transform and its Inverse. Laplace Transform of derivatives. Laplace transform of integrals. Convolution theorem (Statement only). Application to the solution of ordinary differential equations of second order with constant coefficients.

2. Series solution at an ordinary point : Power Series solution of ordinary differential equations, Simple problems only.

Books for Reference

1. Advanced Calculus - David Widder (Prentice Hall)
2. Elementary Treatise on Laplace Transform - B. Sen (World Press).
3. Operational Methods in Applied Mathematics - H. S. Carslaw. J. C. Jaeger.

Group-C (10 Marks)
(Tensor Calculus)

A tensor as a generalized concept of a vector in an Euclidean space E^3. To generalize the idea in an *n*-dimensional space. Definition of E^n. Transformation of co-ordinates in E^n (n = 2, 3 as example). Summation convention.

Contravariant and covariant vectors. Invariants. Contravariant, covariant and mixed tensors. The Kronecker delta. Algebra of tensors Symmetric and skew-symmetric tensors. Addition and scalar multiplication. Contraction. Outer and Inner products of tensors. Quotient law. Reciprocal Tensor. Riemannian space. Line element and metric tensor. Reciprocal metric tensor. Raising and lowering of indices with the help of metric tensor. Associated tensor. Magnitude of a vector. Inclination of two vectors. Orthogonal vectors. Christoffel symbols and their laws of transformations, Covariant differentiation of vectors and tensors.

Books for Reference

1.   Tensor Calculus - Barry Spain
2.   Vector Analysis and Tensor Calculus (Schaum Series) – Spiegel

Paper-VIII B
Practical (50 Marks)

Numerical Analysis*

Newton's forward & backward interpolation. Stirling & Bessel interpolation. Lagrange's and Newton's Divided Difference interpolation. Inverse Interpolation.

Numerical differentiation based on Newton's forward and backward formulae.

Numerical integration : Trapezoidal and Weddle's rule.

Numerical solution of non-linear equations : Tabulation, Bisection, Secant/Regula Falsi and Fixed-point iteration methods.

Numerical solution of a system of linear equations : Gauss elimination method, Gauss Seidal iteration method. Matrix inversion by Gauss method.

Power method for finding the extreme eigenvalues. (3 x 3 or 4 x 4 order).

Statistics *

Sample characteristics - mean, variance, skewness, kurtosis, excess, mode, median, semi-interquartile range. Bivariate samples - correlation coefficient,   regression   lines,   parabolic curve fitting, goodness of fit.

Confidence intervals for mean and standard deviation of a normal population. Approximate confidence limits for the parameter of a binomial populations.

Tests of hypothesis - tests on mean and standard deviation of a normal population, comparison of means and standard deviations of two normal populations. Approximate tests on the parameter of a binomial population, on comparison of two binomial populations. Poisson distribution.

Above problems are to be done on a non-programmable scientific calculator.

The following problems are to be done on computers using either FORTRAN or C compiler:

1. Numerical integration by Simpson's 1/3 rule.

2. Numerical solution of non-linear equation by Newton - Raphson method.

3. Numerical solution of ordinary differential equation by Runge-Kutta (4th order) method.